Week 18 (1/13/03)

Distribution of primes

Let $P(N)$ be the probability that a randomly chosen integer, N, is prime. Show that

$$P(N) = \frac{1}{\ln N}\,.$$

Note: Assume that N is very large, and ignore terms in your answer that are of subleading order in N. Also, make the assumption that the probability that N is divisible by a prime p is exactly $1/p$ (which is essentially true, for a large enough sample size of numbers).

Correction: (Thanks to Bob Silverman for pointing this out.) The assumption, "the probability that N is divisible by a prime p is exactly $1/p$," it is actually *not* valid. More precisely, when dealing with a sufficiently large number of primes, the probability that a prime p divides N is *not* independent of the probability that a prime q divides N. (This is related to Mertens' theorem, which you can look up.) The solution I posted for this problem is therefore incorrect, even though it does end up giving the correct result of $P(N) = 1/\ln N$. (A correction factor ends up canceling out.) However, you might still find it interesting to solve the problem using the given (invalid) assumption.