Week 60 (11/3/03)

Cereal box prizes

In each box of a certain cereal there is a prize, which is one of N colors. Assuming that the prize in each box has equal odds of being any color (even after you've bought many boxes), how many boxes do you need to buy, on average, before you collect all the different colors?

Let $P(n)$ be the probability that you collect the final color in the nth box. For what value of n, in terms of N, is $P(n)$ maximum? Assume that N is large, and ignore terms in your answer that are of subleading order in N.