Week 70 (1/12/04)

Painting a funnel

Consider the curve $y = 1/x$, from $x = 1$ to $x = \infty$. Rotate this curve around the x-axis to create a funnel-like surface of revolution. The volume of this funnel is

$$V = \int_1^\infty \frac{\pi}{x^2}\, dx = \pi,$$

which is finite. The surface area, however, is

$$A = \int_1^\infty \frac{2\pi\sqrt{1+y'^2}}{x}\, dx \;\; > \;\; \int_1^\infty \frac{2\pi}{x}\, dx,$$

which is infinite. So it seems like you can fill up the funnel with paint, but you can't paint it. What is the solution to this apparent paradox?