Week 84 (4/19/04)

Poisson and Gaussian

Throw N balls at random into B boxes. Let a be the average number of balls, N/B, in a box. Let $P(x)$ be the probability that a given box has exactly x balls in it.

(a) Show that

$$P(x) \approx \frac{a^x e^{-a}}{x!} .$$

Certain assumptions are needed for this expression to be valid. What are they?

(b) Show that if a is large, the above Poisson distribution essentially becomes a Gaussian distribution,

$$P(x) = \frac{a^x e^{-a}}{x!} \approx \frac{e^{-(x-a)^2/2a}}{\sqrt{2\pi a}} .$$