Solution

Week 32 (4/21/03)

The game show

If you do not switch, your probability of winning equals 1/3. No actions taken by the host can change the fact that if you play a large number, N, of these games, then (roughly) $N/3$ of them will have the prize behind the door you pick.

If you switch, your probability of winning turns out to be greater. It increases to 2/3. This can be seen as follows. Without loss of generality, assume that you pick the first door. There are three equally likely possibilities for what is behind the three doors: PGG, GPG, and GGP (where P denotes prize, and G denotes goat). If you do not switch, then in only the first of these three cases do you win, so your odds of winning are 1/3. If you do switch, then in the first case you lose, but in the other two you win (because the door not opened by the host has the prize). Therefore, your odds of winning are 2/3, so you do in fact want to switch.

REMARKS:

1. After the host reveals a goat, there is one prize and one goat behind the unopened doors. You might think that this implies that the probability of winning the prize is 1/2, independent of whether or not a switch is made. This is incorrect, because the above reasoning shows that there is only a 1/3 chance that the door you initially picked has the prize, and a 2/3 chance that the other unopened door has the prize. (The fact that there are two possibilities doesn't mean that their probabilities have to be equal, of course.)

2. It should be no surprise that the odds are different for the two strategies *after* the host has opened a door (the odds are obviously the same, equal to 1/3, whether or not a switch is made *before* the host opens a door), because the host gave you some of the information he had about the locations of things.

3. To make the above reasoning more believable, imagine a situation with 1000 doors (containing one prize and 999 goats). After you pick a door, the host opens 998 other doors to reveal 998 goats. In this setup, if you do not switch, your chances of winning are 1/1000. If you do switch, your chances of winning are 999/1000 (which can be seen by listing out the 1000 cases, as we did with the three cases above). In this case it is clear that the switch should be made, because the only case where you lose after you switch is the case where you had initially picked the prize (and that happens only 1/1000 of the time).

4. The clause in the statement of the problem, "The host announces that after you select a door (without opening it), he will open one of the other two doors and reveal a goat," is crucial. If it is omitted, and it is simply stated that, "The host then opens one of the other doors and reveals a goat," then it is impossible to state a preferred strategy. If the host doesn't announce his actions beforehand, then for all you know, he *always* reveals a goat (in which case you should switch). Or he *randomly* opens a door, and just happened to pick a goat (in which case it doesn't matter whether or not you switch, as you

can show). Or he opens a door and reveals a goat if and only if your initial door has the prize (in which case you definitely should not switch).

5. This problem is infamous for the intense arguments it so easily lends itself to. The common incorrect answer is that there are equal 1/2 chances of winning whether or not you switch. Now, there's nothing bad about getting the wrong answer, nor is there anything bad about not believing the correct answer for a while. But concerning the arguments that drag on and on, I think it should be illegal to argue about this problem for more than ten minutes, because at that point everyone should simply stop and *play the game*. Three coins with a dot on the bottom of one of them is all you need. Not only will the actual game give the correct answer (if you play enough times so that things average out), but the patterns that form when playing will undoubtedly convince the skeptic of the correct reasoning. Arguing endlessly about an experiment, when you can actually *do* the experiment, is as silly as arguing endlessly about what's behind a door, when you can simply *open* the door.