AN EXPERIMENTAL LIMIT ON PROTON DECAY:

\[ p \rightarrow e^+ + \pi^0 \]

A Thesis Presented
by
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ABSTRACT

A search for the decay $p + e^+ \pi^0$ was performed in the 8000 metric ton Irvine-Michigan-Brookhaven water Cherenkov detector which was constructed underground at a depth of 1570 meters of water equivalent. In a 130 live day exposure ($3.9 \times 10^{32}$ proton years in the 3300 ton fiducial volume of the detector) no events consistent with the above decay were found. With calculated detection efficiencies of 0.92 and 0.54 for $p + e^+ \pi^0$ events occurring in hydrogen and oxygen respectively, this thesis establishes at the 90% confidence level that the partial lifetime for this decay mode exceeds $1.1 \times 10^{32}$ years. This result is incompatible with predictions from minimal SU(5), the simplest of the Grand Unified Theories and with previously reported experimental measurements. For $p + e^+ \pi^0$ decays occurring from free protons in the hydrogen in our detector we establish a partial lifetime limit which is independent of nuclear effects of $3.1 \times 10^{31}$ years. The rate and characteristics of neutrino interactions observed in our detector (112 events in 130 days) are compatible with expectations.

This thesis documents the author's design for the electronics readout system for the IMB detector as well as his event fitting, reconstruction, and event simulation software.
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Profs. Frank Pipkin and C. Papaliolios kindly consented to stand in for Larry as my advisor(s) at Harvard, and I thank them here. Prof. Karl Strauch (Chairman of the Physics Department in 1979) was a model of academic impartiality and fairness in the difficult political situation of the time, for which I shall always be grateful.

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Keeping promises made several times to myself, two more acknowledgements are due: to R. Spehar of Norris Bros. Construction Co. for the comaraderie that develops in weeks of working together on I-beams sixty feet off of the ground; and to Eric Hazen for bleeding all over the electronics with me.
Chapter 1 - INTRODUCTION

I. Conservation of Baryon Number

The apparent stability of the proton and bound neutron against the large number of decay modes allowed by the conservation laws of energy, electric charge, and angular momentum is a longstanding mystery of particle physics. This led to the hypothesis [1] of a conserved 'Baryonic Current', analogous to electric current, and a related conserved baryonic charge $B$ (the number of protons + neutrons + baryonic resonances minus the number of their antiparticles). Like electric charge, $B$ has been found to be conserved in all known interactions. Unlike electric charge, however, $B$ does not seem to be associated with the gauge symmetry of a long-range force of any appreciable strength [2], giving rise to recurrent speculations [3] that the conservation of $B$ may be only approximate.

Within the confines of the SU(3) x SU(2) x U(1) 'Standard Model' [4] of particle physics, baryon number conservation is understood as a reflection of the fact that quarks and leptons occur in different multiplets. This is true separately for each of the gauge symmetry factor groups. Since the gauge bosons associated with each of the symmetry groups of the standard model induce fermion-number conserving transitions only between members of the same
multiplets, no mechanism exists for changing either the quark number or the lepton number L of an initial state. (Each quark is assigned a baryon number B=1/3 so that baryons are qqq bound states with B=1). Thus B (and L) conservation can be viewed not as fundamental laws but as 'accidental' occurrences due to the choice of fermion representations in the standard model.

Cosmological arguments also suggest that B (and L) may not be absolutely conserved. The observed matter domination of the universe may have evolved from a matter-antimatter symmetric initial state through the CP violating decays of heavy bosons during a departure from thermal equilibrium in the early moments of the Big Bang [5]. This requires violation of B, L, and CP - all of which are naturally occurring ingredients in Grand Unified Theories.

II. Grand Unified Theories

The success of the SU(2)xU(1) unified theory of the electromagnetic and weak interactions[6] led to immediate interest in including the interactions of the standard model into a single 'Grand Unified' gauge group $G_{GUT} \subseteq SU(3) \times SU(2) \times U(1)$. In analogy to the electroweak theory, $G_{GUT}$ is expected to be a badly broken symmetry at experimentally accessible energies. It would presumably be exact only for experiments carried out at energies much greater than the energy (mass) scale of the symmetry
breaking.

It was realized early that any such an embedding would place quarks and leptons into common multiplets, and that the gauge bosons mediating transitions between them would almost inevitably lead to nucleon decay [7]. The observed stability of the proton could be explained if the mass $M_X$ of the gauge bosons were sufficiently large. By dimensional analysis, the proton lifetime $\tau_p \sim M_X^4$. The electroweak theory again provides an analogy: The approximate conservation of strangeness (strange quark number) would be exact in the limit $M_W, M_{Z0} \rightarrow \infty$. The experimental observation of the violation of strangeness (and of baryon number) depends critically on the mass of the gauge bosons involved.

The simplest Grand Unified Theory is the 'minimal' SU(5) theory invented by Georgi and Glashow in 1974 [8]. The term 'minimal' refers to the property of the model that the size of the gauge group, the pattern of symmetry breaking (i.e. the number of Higgs bosons), and the fermion representations are all the simplest allowed by the phenomenology of the standard model.

In general the coupling constants of the three gauge groups of the standard model are restricted by the embedding in the grand unified gauge group. This often leads to experimentally testable predictions. The simplest
prediction is that the ratios of the electric charges of the quarks and leptons cannot be arbitrarily fixed (as in the standard model), but must be the ratios of small integers determined from group theory (e.g. \( Q_q/Q_1 = \pm 2/3 \) or \( \pm 1/3 \) for the representations chosen for minimal SU(5)).

The second experimentally testable prediction of the SU(5) model is the calculation of the weak mixing angle \( \theta_w \) which is related to the ratio of the SU(2) and U(1) coupling constants \( g_2 \) and \( g_1 \) by

\[
\tan \theta_w = g_1/g_2.
\]

The requirement that the SU(2) and U(1) generators be equivalently normalized generators of the SU(5) group means that \( g_1 \) and \( g_2 \) are not independent, but are given by

\[
\sin^2 \theta_w = 3/8 \quad (E \gg M_x).
\]

This result is valid only at energy scales above those at which the SU(5) symmetry is broken. Taking into account the different energy dependences of the SU(2) and U(1) coupling constants, one obtains the experimentally confirmed predictions

\[
\sin^2 \theta_w = 0.214 \pm 0.004 \quad \text{(theory) [10]},
\]

vs.

\[
\sin^2 \theta_w = 0.215 \pm 0.014 \quad \text{(experiment) [11]}.
\]

A third experimentally testable prediction of SU(5) is the lifetime for decay of protons and bound neutrons.
III. Nucleon Decay Rates and Branching Ratios in SU(5)

Nucleon decay occurs at some level in almost all Grand Unified Theories, however, minimal SU(5) is unique in predicting the absolute decay rate as well as approximate branching ratios. The crucial element in the calculation is the determination of the mass $M_x$ of the gauge bosons (Fig. 1.III.1) which mediate the decay. This is done using the experimentally measured values of the gauge coupling constants $g_1$, $g_2$, and $g_3$ and their theoretically known $Q^2$ dependence. $M_x$ is determined by requiring that the coupling constants attain the values predicted by SU(5) at $Q^2 \sim M_x^2$, as indicated in Fig. 1.III.2. In principle the relations between the three coupling constants yield two independent predictions for $M_x$. In common practice, $M_x$ is determined using experimental measurements of the QCD scale parameter $\Lambda$, with the other relation turned around to 'predict' $\sin^2 \theta_W$ as a check on the theory. For the SU(5) model $M_x$ is determined to be $\sim 10^{15}$ GeV.

The difficulty with this calculation is that the error bars from the extrapolation of the low-energy coupling constants are taken to the 4th power in determining the proton lifetime. This leads to a range in the theoretically predicted lifetimes of

$$\tau_p \equiv 2 \times 10^{(29 \pm 1.7)} \text{ years} \ [10].$$
Fig. 1.III.1. Diagrams contributing to $p + e^+\pi^0$ in SU(5).
Fig. 1. Determination of $M_X$ using the $Q^2$ dependences of the SU(3), SU(2), and U(1) coupling constants. The theoretical extrapolation from their experimentally determined values at low energies yields two independent estimates of $M_X$. 
Thus any definitive experimental test of these theories should be sensitive to lifetimes $\tau > 10^{31}$ years.

Branching ratios for nucleon decay in the SU(5) theory have been calculated under a variety of simplifying assumptions by a number of different authors [12]. Fig. 1.III.3 (taken from Ref. [13]) is representative of these calculations. Note that the branching ratios $B$ for $p + e^+\pi^0$ (35-40%) and $N + e^+\pi^-$ (65-75%) are large and essentially model-independent. The decay rates for protons and neutrons in the SU(5) model are related by a $\Delta I=1/2$ rule [14] which implies e.g. $\Gamma(p + e^+\pi^0) = 1/2 \Gamma(n + e^+\pi^-)$.

This thesis will examine the decay $p + e^+\pi^0$ rather than the decay $n + e^+\pi^-$ because of the experimental difficulties associated with the strong interactions of charged pions. The partial lifetime for $p + e^+\pi^0$ is predicted to lie in the range

$$\frac{\tau}{B(e^+\pi^0)} = 4.5 \times 10^{29(\pm 1.7)} \text{ years} [10].$$

IV. Previous Experimental Results

Motivated by the general considerations of baryon stability outlined in Sect. 1.I, a long series of searches raised the experimental limits on the nucleon decay lifetime from $\sim 10^{21}$ years (Reines, Cowan, and Goldhaber, 1954 [15]) to $\sim 3 \times 10^{30}$ years (Reines and Crouch, 1974 [16]). The most sensitive of these searches directly monitored matter looking for the charged decay products of nucleons, whereas
\[ \text{TABLE I. Branching ratios for proton decay in SU}_3 \text{ in three kinematic models.} \]

<table>
<thead>
<tr>
<th>Channel</th>
<th>Static model</th>
<th>Recoil model</th>
<th>Relativistic model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+\omega$</td>
<td>21.4%</td>
<td>24.9%</td>
<td>25.9%</td>
</tr>
<tr>
<td>$e^+\rho^3$</td>
<td>2.4%</td>
<td>6.6%</td>
<td>10.5%</td>
</tr>
<tr>
<td>$e^+x^3$</td>
<td>35.7%</td>
<td>39.8%</td>
<td>38.4%</td>
</tr>
<tr>
<td>$e^+\eta_8$</td>
<td>6.9%</td>
<td>1.5%</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_0^+$</td>
<td>1.0%</td>
<td>2.6%</td>
<td>4.3%</td>
</tr>
<tr>
<td>$\rho_0^+$</td>
<td>14.3%</td>
<td>15.9%</td>
<td>15.4%</td>
</tr>
<tr>
<td>$\mu^+K^0$</td>
<td>18.3%</td>
<td>8.4%</td>
<td>4.9%</td>
</tr>
<tr>
<td>$\nu_\alpha K^+$</td>
<td>0</td>
<td>0.2%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Total</td>
<td>100.1%</td>
<td>99.9%</td>
<td>99.9%</td>
</tr>
</tbody>
</table>

\[ \text{TABLE II. Neutron-decay branching ratios in SU}_3 \text{ in three models.} \]

<table>
<thead>
<tr>
<th>Channel</th>
<th>Static model</th>
<th>Recoil model</th>
<th>Relativistic model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu\omega$</td>
<td>4.7%</td>
<td>4.3%</td>
<td>4.6%</td>
</tr>
<tr>
<td>$\nu\rho^3$</td>
<td>0.6%</td>
<td>1.2%</td>
<td>1.8%</td>
</tr>
<tr>
<td>$\nu x^3$</td>
<td>7.8%</td>
<td>7.3%</td>
<td>6.8%</td>
</tr>
<tr>
<td>$\nu \eta_8$</td>
<td>1.5%</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$e^+\rho^-$</td>
<td>5.5%</td>
<td>11.8%</td>
<td>18.3%</td>
</tr>
<tr>
<td>$e^+\tau^-$</td>
<td>78.7%</td>
<td>72.2%</td>
<td>67.3%</td>
</tr>
<tr>
<td>$\nu_\alpha K^0$</td>
<td>1.1%</td>
<td>3.0%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Total</td>
<td>99.9%</td>
<td>100.0%</td>
<td>100.1%</td>
</tr>
</tbody>
</table>

Fig. 1.III.3. Calculated branching ratios for proton and neutron decay in SU(5). Taken from Ref. [13].
geochemical searches [17] yielded limits which were not as stringent but less dependent upon the assumed decay mode.

When the specific predictions for nucleon decay at a level of $10^{27}$-$10^{32}$ years emerged from SU(5) and other Grand Unified Theories [18] , a number of new searches were initiated.

The first new results were obtained from underground neutrino detectors which were converted for use in the search for nucleon decay. In 1981 Cherry et. al. [19] , using a segmented water Cherenkov detector, obtained a limit of $2.4 \times 10^{31}$ years against nucleon decay modes which ultimately yield a stopping $\pi^+$ or $u^+$. This was a factor of three improvement over the limits set on the basis of a re-analysis of the Reines data[20]. The Kolar Gold Field detector started taking data in 1981, and in 1982 reported a signal from nucleon decay corresponding to a lifetime of $\sim 7 \times 10^{30}$ years [21]. They claimed three fully contained candidate events, including one event ascribed to $p + e^+\pi^0$ in an exposure of $\sim 60$ tons x 1.5 years = 90 ton-years. The detector consists of a series of 1.2 cm iron plates interleaved with proportional tubes with 10 cm spacing, so that each track in an event is characterized by 2-3 firings in each of two orthogonal views. No timing information is available, so that the location of the primary vertex of the events may be uncertain. The neutrino background to the
events is claimed by the authors to be small.

The first dedicated nucleon decay experiment to report results was the Mont Blanc NUSEX collaboration [22]. In an exposure of 40 ton-years, they report a single event which was consistent with the proton decays modes \( p + \mu^+\bar{\nu} \), \( p + K^0 \), or \( p + \mu^+\mu^-\mu^+ \), with an estimated background of \(<.01\) events. In view of the branching ratios of the Fig. 2.I.3 above, this event is an unlikely first candidate from the minimal SU(5) model, although \( p + \mu^+\bar{\nu} \) may occur if the decay rate is dominated by the interactions of superheavy Higgs particles [23].

In early 1983 the IMB collaboration published a limit on the decay \( p + e^+\pi^0 \) [24] which is the subject of this thesis. In 80 live days (a fiducial volume exposure of 725 ton-years, \( \sim 8x \) that of the KGF result), no candidate events were found. This thesis reports an extension of that limit to 130 analyzed live days.
Chapter 2 - THE EXPERIMENT

"I only work on experiments that show results".
- Carlo Rubbia to Tegid Jones, January 1983

I. Detector Overview: Response to $P + e^+\pi^0$

The IMB Nucleon Decay Detector (Fig. 2.I.1) is a rectangular volume of water approximately $22 \times 17 \times 18$ m in size which is surrounded on its six faces by 2048 photomultiplier tubes (PMT's) each of which are $12.7$ cm in diameter. The tubes are capable of detecting single photoelectrons from Cherenkov radiation from charged particles moving within the detector at $\beta > 0.75$. The detector can resolve the time structure of the Cherenkov wave front and thereby determine the vertex position of an event as well as the direction of energy flow of tracks away from the vertex.

The PMT's are inset ~50 cm from the detector walls and face inwards. The fiducial volume of the detector is a software-defined region inset 2 m from the planes of the PM tubes, which provides for an ~2.5 m active veto region on all sides of the fiducial volume.

Fig. 2.I.2 illustrates the detector response to an idealized two-body nucleon decay event. The photograph
Fig. 2.1.1. Schematic view of the IMB nucleon decay detector, which consists of a rectangular volume of water surrounded on its surface by 2048 hemispherical photomultiplier tubes which record the Cherenkov light emitted from relativistic charged particles.
Fig. 2.I.2. Detector response to idealized two-body nucleon decay. This is a cross-sectional view, with the positions of the PM tubes indicated near the edges of the water volume. Two back-to-back charged tracks ~2.5m long emit Cherenkov photons which travel outward in a pair of cones at an angle of 41°. The resultant pattern of PM firings is a pair of back-to-back rings on opposite walls of the detector, with a timing pattern approximating that of a spherically expanding wavefront from a point source of light.
represents a two dimensional cross section of the detector, with the positions of the tubes indicated near the edge of the water volume. A nucleon decays at rest into two back-to-back charged tracks, each with ~0.5 GeV of energy and which lose energy in the water at a rate of 2 MeV/cm until they stop after 2.5 m. The Cherenkov photons are emitted in a three-dimensional cone at an angle of <41° from each track. They continue outwards until they strike the wall of PMT's, where their position, pulse height, and time of arrival are measured and recorded. Owing to the ~20m size of the detector, the time differences are large (scores of nanoseconds) and easy to measure. The resultant pattern of ~175 tube firings is a pair of back-to-back rings on opposite walls of the detector, with a timing pattern consistent with that produced by a point source of light in the interior of the detector. The series of opposing timing measurements (top vs. bottom, east vs. west in the photo) provide a means of determining the event vertex within σ~50 cm for artificial light sources and simulated nucleon decays.

The total mass inside the planes of the phototubes of the detector is 7000 metric tons of chemically pure water. The fiducial volume of the detector is a rectangular region inset 2.0 m from the planes of the phototubes of 3300 metric tons (2.0 x 10^{33} nucleons). Events whose vertex reconstructs inside this volume are considered "contained"
events, and estimates of the detection efficiency, neutrino interaction rates, etc., are computed on the basis of this volume.

Fig. 2.I.3 indicates some of the important physical processes which smear the detector response to $p + e^+\pi^0$. These include the following:

(i) $\pi^0 \rightarrow \gamma\gamma$ decay kinematics following $p + e^+\pi^0$ which separate the two photons by an angle of typically $40^\circ$. The two showers from $\pi^0$ decay $\gamma$'s are generally not resolvable due to the overlap of the Cherenkov cones. Since most of the $\pi^0$ energy is carried away by the photon which is boosted forward along the direction of motion of the $\pi^0$, the average direction for emission of Cherenkov radiation remains close to the initial $\pi^0$ direction independently of the decay angle in the $\pi^0$ rest frame.

(ii) Shower development in water (radiation length $X_0 = 36$ cm) for the $e^+$ and $\gamma$'s from $\pi^0$ decay. This introduces fluctuations about the mean angle for emission of Cherenkov photons, which to some extent obscures the structure of the rings. Since the shower size $\sim 7X_0 = 2.5$ m in water, most $p + e^+\pi^0$ decays occurring in the fiducial volume will be fully contained inside the planes of the tubes so that an accurate calorimetric measurement can be made.

(iii) The fluctuations in the total Cherenkov radiating
Fig. 2.1.3. Electromagnetic showers in water following $p + e^+ \pi^0$ decays of free protons in hydrogen. These are four case histories from computer simulations of the decay kinematics. The EGS routines [38] are used to simulate shower development in the water.
tracklength are small (< 1% for a 0.5 GeV shower), so that the detector shower energy resolution is limited by sampling fluctuations, and in particular by the Poisson fluctuations of ~10% in the collection of ~110 photoelectrons per 0.5 GeV track. Since the fractional coverage of photocathode (1.5%) is approximately constant when averaged over the solid angle surrounding an event, the amount of collected light is insensitive to the track angles and position of the event within the fiducial volume of the detector.

(iv) Light scattering in the water (both Rayleigh and Mie scattering are important) further smears the detector response by redistributing light outside the Cherenkov cone. Approximately 15% of the light collected from an event has been scattered at least once before detection. Due to the longer pathlength of isotropically scattered light, the majority of scattered light is significantly delayed with respect to the rest of the Cherenkov wavefront and can be removed (for the purposes of event reconstruction) by eliminating late PMT firings.

(v) Phototube dark noise of ~2.8 kHz/tube will contribute a mean of ~1 noise firing within ±100 nsec of an event. This is a negligible noise source compared to the effects of light scattering.

(vi) PMT timing resolution varies from 5.5 ns HWHM (~1.3 m in water) at the single photoelectron illumination level to 1.5 ns HWHM at high light levels. The large number of
tubes firing in an event provide a high level of redundancy in the timing measurements, and the timing information is a fundamental tool for vertex localization.

The physics smearing for free $p + e^+\pi^0$ from the $2.2 \times 10^{32}$ free protons from hydrogen in the fiducial volume of our detector will be limited to (i)-(vi) above. For these decays, the back-to-back, 500 MeV per track nature of the decays will be clearly visible. We find that we are able to reconstruct the angle of simulated free $p + e^+\pi^0$ decays as $180\pm15^\circ$. For proton decays occurring in oxygen the following nuclear effects must be considered (see Fig. 2.I.4):

(vii) Fermi motion of the decaying nucleon, which destroys the exact back-to-back nature of the decay by $\sim 20^\circ$. The Lorentz boost due to Fermi motion of one secondary is compensated for by the retardation of the other, so that the net effect is to redistribute the energy between the $e^+$ and $\pi^0$ as much as $\pm 100$ MeV without changing the visible energy of the decay. This is especially true for $p + e^+\pi^0$ in a Cherenkov detector since both secondaries appear as electromagnetic showers, which are efficient radiators of Cherenkov light.

The effects of Fermi motion are larger than the resolution of the detector in track opening angle ($\sim 15^\circ$) and energy balance between the tracks ($\Delta E/E \sim 10\%$ at 1 GeV) so that the $p + e^+\pi^0$ events from oxygen will be observably degraded.

(viii) The binding energy of the most loosely bound
Fermi Motion

$\pi^0$ scattering in nucleus

Charge exchange scattering in nucleus:
$p + e^+\pi^0, \pi^0 + p + \pi^+ + n$

Nuclear absorption of $\pi^0$

Fig. 2.I.4. Physical smearing of $p + e^+\pi^0$ in oxygen.
nucleon in oxygen is 7 MeV. Decays of more deeply bound nucleons may be missing up to 35 MeV of energy [25]. In addition, the recoil kinetic energy carried away by the nucleus as a result of Fermi motion may subtract several MeV from the visible energy of an event. All of these effects are small compared to the shower energy resolution (~10% at 1 GeV) of the detector.

(ix) Nuclear interactions of the π⁰, which can undergo elastic, inelastic, or charge exchange scattering in the oxygen nucleus, or be totally absorbed. Thus any event with a single 0.5 GeV showering track is a potential candidate for p + e⁺π⁰ in oxygen. However, the background from single track neutrino interactions is such that a much more sensitive search can be made by restricting oneself to the ~50% of events for which the back-to-back, equal energy signature can be reconstructed.

The energy measured by the detector is determined by the fundamental properties of Cherenkov radiation:

\[ \theta_C = \cos^{-1}(1/\beta n) \]  
(in water the index of refraction n=1.33)  
+ 41° as \( \beta \rightarrow 1 \)  
+ 0° as \( \beta \rightarrow \beta_{\text{threshold}} = 0.75 \)

and

\[ \# \text{ Photons/cm track/nm wavelength} = 2\pi \alpha \sin^2 \theta_C / \lambda^2 \]  
+ const as \( \beta \rightarrow 1 \)  
+ 0 as \( \beta \rightarrow \beta_{\text{threshold}} = 0.75 \).
A $\beta=1$ charged particle in water yields 770 photons/cm in the range from 300 to 600 nm. The above formulae, when combined with range-energy relationships [26] give the Cherenkov light yield as a function of energy for various particles shown in Fig. 2.I.5. The light yields from showering particles ($e^\pm$, $\gamma$, or $\pi^0$) are essentially equal, and are proportional to the energy $E$. This is true for $E \gg 0.26$ MeV which is the Cherenkov threshold kinetic energy for electrons in water. Heavier charged particles ($\mu, \pi, K, p$) are invisible to the detector below kinetic energy thresholds of 54, 72, 250, and 480 MeV respectively, and the response becomes linear with energy only for $\gamma > 2$. We define two energies: the electromagnetic shower energy

$$E_C = E_{\text{Cherenkov}} = \text{the total event energy calculated from the Cherenkov light yield under the assumption that the event was a showering track, after correction for light attenuation, detector systematics, etc.}$$

and the minimum event energy, after correction for $\mu^\pm$ or $\pi^+$ below threshold (see Fig. 2.I.5),

$$E_{\text{min}} = \begin{cases} E_C, & \text{if no } \mu + e \text{ decay signature was detected, or} \\ E_C + 235 \text{ MeV for each identified } \mu + e \text{ decay.} \end{cases}$$

The efficiency for detecting decay electrons (Sect. 2.V.F) from stopping $\mu^+$ and $\pi^+$ is 65% and for $\mu^-$ is 60%.
Fig. 2.I.5. Cherenkov light yield $E_C$ vs. total energy $E$ for various particles. $E_C$ is equal to the total energy for showering tracks.
The different sensitivity for various particle types is an important factor in evaluating the neutrino background as well as the sensitivity to various nucleon decay modes. In particular, the detection efficiency for low energy $v_e$ events is much greater than for $v_\mu$ events. The protons and heavy charged nuclear breakup products from $v$ interactions are essentially invisible.

A summary of the properties of the detector is given in Fig. 2.I.6.

II. **BACKGROUNDs TO $p + e^+\pi^0$**

A. **Single straight-through muons**

The 2000' rock overburden of our detector provides shielding against cosmic rays equivalent to 1570 m. of water. This completely absorbs the electromagnetic and hadronic component of the surface cosmic ray flux and reduces the flux of muons by a factor of $\sim 10^6$, resulting in an average muon trigger rate of 2.7 Hz. The depth dependence of the muon rate and comparative rates in other nucleon decay experiments is indicated in Fig. 2.II.A1.

Although this flux of muons represents a nuisance in data handling, it is a useful tool for monitoring the detector and provides an absolute energy calibration. It is not a background to the $p + e^+\pi^0$ decay mode search reported in this thesis. The reason for this is that an entering charged track must signal its presence by emitting Cherenkov
Fig. 2.I.6. Summary of IMB Detector Properties

**Detector Size:**

- Total mass: 8000 metric tons
- Fiducial Mass: 3300 metric tons
  \[ = 2.0 \times 10^{33} \text{ nucleons} \]

**Energy Resolution:**

- 500 MeV shower: \( \sigma = 13\% \)
- 300-500 MeV/c \( \mu^\pm \): \( \sigma = 15\% \)
- \( p + e^+\pi^0 \): \( \sigma = 10\% \)
- Systematic error in absolute energy calibration: \( < 15\% \)

**Trigger Threshold:**

- Hardware: \( \sim 30 \text{ MeV} \)
- Software: \( \sim 160 \text{ MeV} \)

**Vertex Localization:**

- Nucleon decay: \( \sigma = 65 \text{ cm} \)
- \( \sim 500 \text{ MeV} \nu_\mu \): \( \sigma = 75 \text{ cm} \)
- Cosmic Ray \( \nu \): \( \sigma = 135 \text{ cm} \)

**Angular Resolution:**

- Showering tracks: \( \sigma = 6^\circ \)
- 300-500 MeV/c \( \mu^\pm \): \( \sigma = 4^\circ \)
- Straight-through muons: \( \sigma < 4^\circ \)

**Track Direction:**

- No sign ambiguity

**\( \mu + e \) Detection:**

- \( \varepsilon \sim 65\% \) after trigger (\( \mu^+ \))
- Several \( \mu + e \) ok
- \( e \) direction: \( \sigma = 25^\circ \)
- \( e \) energy: \( \sigma = 40\% \)
Fig. 2.II.A1. Muon trigger rate vs. depth (adapted from reference [50]).
light for a minimum of 2.5 m before it enters the fiducial volume of the detector. Thus any entering track with a radiating pathlength comparable to the ~5m length expected for proton decay will have deposited the majority of its light in the active veto region of the detector, and can quickly and easily be rejected.

The detector response to a cosmic ray muon is illustrated in Fig. 2.II.A2. The PMT's fire in a characteristic sequence as the Cherenkov wavefront sweeps through the detector, indicating the timing, topological, and calorimetric information used to identify this as a background event.

Fig. 2.II.A3 displays the muon energy spectrum at our depth for various elevation angles. It is characterized by (i) a mean muon energy of ~200 GeV, (ii) its flatness out to ~100 GeV, and (iii) its approximate $\cos^3$ dependence on zenith angle. The rate and angular distributions of the muons passing through our detector have been measured [26] and found to be in agreement with results obtained at our depth by previous cosmic ray experiments [27]. Detailed parameterizations of the muon angle and energy spectrum are given in Appendix B.1. A single muon passing vertically through our detector deposits $E_c \sim 4.8$ GeV of Cherenkov equivalent energy and hence does not constitute a background to nucleon decay. About 30% of the muons pass
Fig. 2.II.A2. Detector response to straight-through muon. The Cherenkov wavefront sweeps through the detector, illuminating in sequence: (a) the 'entry signature' of 3-5 PMT's near entry point, (b) tubes along the side wall, (c) bottom tubes illuminated at high pulse height near the exit point, and (d) all tubes inside the filled-in Cherenkov cone.
Fig. 2.II.A3. Muon energy spectrum at 1600 meters of water equivalent, for different zenith angles. Note the flatness out to $E_\mu \sim 100$ GeV. The dependence on zenith angle is approximately $\cos^3 \theta_z$. 
through a smaller region of the detector and deposit Cherenkov energy within the $0.2 \text{ GeV} + 1.8 \text{ GeV}$ range in which we record $\nu$ interactions in the detector. About 10% of these "corner clippers" will deposit $E_C = 940\pm30\%$, the range in which we search for $p + e^+\pi^0$. These are rejected using the timing and topology information from the light emitted as they enter through the veto region of the detector.

B. Stopping Muons

At the depth of our detector stopping muons arise from three sources:

(1) Single muons which simply range out in our detector and which constitute ~4% of all triggers. Those which pass the initial $0.2-1.8 \text{ GeV}$ energy cut are a relatively important fraction of the events which survive the early and simple stages of the analysis. However, they are very easily fit as single clean tracks in later stages of the analysis since they do not interact strongly and multiple scattering becomes important only below Cherenkov threshold.

(2) Locally produced stopping muons from $\pi \rightarrow \mu$ decays in hadronic showers initiated by the photonuclear interactions of nearby muons, which will be discussed in Sect. D below. These are generally accompanied into the detector by the parent muon and hence the majority of these are easily rejected.

(3) Neutrino induced stopping muons from charged-current
interactions in the surrounding rock, which are negligible in comparison to (1).

C. **Multiple Muons**

Although depressed in rate by a factor of \( \sim 50 \) relative to single muons, multiple muon events represent a potentially pathological background due to the possibility of 2 track topology. These muons are independently produced in large hadronic showers from energetic primary protons (or heavy nuclei at an equivalent energy per nucleon) as they hit the upper atmosphere. They pass through the detector as an essentially isochronous (time spread < 1 ns) wavefront perpendicular to their direction of motion, and are separated laterally by the following effects [29]:

1. Transverse momenta of 250–450 MeV from their production point in the upper atmosphere, which at our depth results in a radial width of \( \sim 6 \) m for muon 'bundles' originating from a common primary.

2. Magnetic field separation of opposite-sign muons, which depends on muon angle and energy and which results in muon separations of typically 3 m.

3. Multiple scattering of the muon in the rock, which depends on \( E_\mu \) and slant depth but which results in separations of 0.5–1.5 m and is less important than (1)–(2) above.
The survival of multiple muons through various stages of data analysis will be discussed in Sect. 3.II ff.

D. Muon Associated Backgrounds

The cross sections for various electromagnetic interactions of muons [Appendix B.2] are such that 1 muon in 7 which passes through our detector suffers a single electromagnetic collision releasing more than 1 GeV of energy. This could in principle result in a source of background due to, e.g., isolated $\gamma$'s which enter the detector from muons passing through the nearby rock. In practice this does not present a problem for the following reasons:

- The event can be vetoed if the parent muon passes through the detector.

- Due to the kinematics of the scattering, any $E>1$ GeV shower is initiated at a very small angle ($<1^\circ$) with respect to the parent muon and the transverse development of the shower is small ($<50$ cm). This means that only very specific muon and shower topologies will send unaccompanied $e^+, e^-, \gamma$ fragments into the detector.

- Entering shower fragments must pass through a minimum of 7 radiation lengths of water before entering the fiducial volume of the detector. Any shower which deposits a significant energy in the fiducial volume is likely to produce a much larger signal in the active veto region,
resulting in an event which appears as a 'flash of light' in the corner of the detector, and which is easily rejected by the point fit (Sect. 3.III).

Photonuclear (deep inelastic) interactions will produce hadronic showers approximately once per 200 muons passing through our detector. The possibility of isolated neutrons entering the detector from these interactions has been studied in Refs. [30] which find as expected that isolated neutrons interact near the edge of the detector and that the absolute rate is small even before imposing the requirement that they mimic the back-to-back topology of 2 body nucleon decay.

A concluding general observation is that all muon associated backgrounds should signal their presence as contamination of the 'contained event' sample by an excess of downward-going events near the top and side walls of the detector. As discussed in Sect. 4.I, we have at our current level of statistics no evidence for such a contamination.

E. Neutrino Backgrounds

We observe about 1 neutrino interaction per day inside the fiducial volume of our detector. A 1 GeV neutrino has an interaction probability of $\sim 10^{-4}$ as in passing through the Earth, so that these interactions will be distributed homogeneously throughout the detector and be nearly isotropic in direction. Since neutrinos do not signal their
entrance into the detector as do relativistic charged particles, they are a source of apparently contained events which is independent of depth and form the ultimate background to nucleon decay searches.

The ambient neutrino flux arises in principle from 4 sources:

(i) Solar ν's, which have a maximum energy of ~14 MeV and are therefore negligible as backgrounds to e⁺π⁰ proton decay. Neutrinos from natural radioactivity are likewise negligible.

(ii) Gravitational collapse of stars at a distance <~20 kpc may produce a burst of neutrinos which may deposit > 1 GeV of energy in our detector [31]; however, the time scale of the burst is ~1 sec and the individual neutrino energies are only 10-20 MeV and are thus negligible.

(iii) Extraterrestrial neutrinos may be produced by the interactions of cosmic ray primaries with interstellar or intergalactic gas, stellar atmospheres, etc. A limit on this flux may be set from observations of the flux of >~100 MeV γ's (arising from π⁰'s which would be produced in numbers comparable to ν's), which yields a flux ~10⁴ times lower[32] than (iv) below.

(iv) Terrestrial (atmospheric) neutrinos and antineutrinos, produced in hadronic showers following the
interactions of cosmic ray primaries in the upper atmosphere, are apparently the only significant source of neutrinos in our detector. They arise primarily via the decays in flight:

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu + e^\pm + \nu_\mu + \nu_e$$

and

$$K^\pm \rightarrow \mu^\pm + \nu_\mu$$

The flux of $\tau$ neutrinos is negligible.

Calculations of the flux of muon and electron neutrinos and antineutrinos have been performed by a number of authors [33] (see Fig. 2.II.El) and will be discussed further in Sect. 2.IV and Appendix B.8. The calculations are considered to be accurate to $\pm 15\%$ for neutrino energies above 2 GeV, while below 1 GeV the uncertainty could be as much as $\pm 50\%$. Disagreement between different calculations is at the 30\% level (see Fig. 2.II.El). Thus the absolute rate of neutrino interactions below 1 GeV is insufficiently well known, e.g., to make a background subtraction; however, our observation of neutrino interactions at approximately the correct rate serves as an important experimental benchmark.

The majority of $\nu$ interactions at our energies are charged current 'quasi-elastic' events (Fig. 2.II.E2(a))
Fig. 2.II.E1. Cosmic ray neutrino spectra calculated by various authors.
Fig. 2.II.E2. Neutrino event topologies.
(a) charged-current quasi-elastic
(b) single pion production
(c) multipion production
which will have 1 charged lepton ($\mu^+$ or $e^+$) which may be visible in our detector; the recoiling hadronic system is below Cherenkov threshold. These single track events are useful for observing the general properties of $\nu$ interactions and verifying the detector's ability to identify contained events, but do not mimic the two-track, equal energy per track, back-to-back proton decay signature of this search.

Single and multiple pion production (Fig. 2.II.E2(b), (c)) becomes significant for $E_\nu \sim 1$ GeV. Multipion production dominates only at $E_\nu \sim 2$ GeV, and the momentum imbalance in the initial state is such that the general topology of the events is a spray of particles in the direction of the incident neutrino. Single pion production occurs primarily through $\Delta$ resonances and is a much more serious background due to the possibility that the pion and the lepton may occasionally be emitted in nearly the back-to-back configuration expected for $p + e^+\pi^0$. Roughly speaking, a neutrino simulating $p + e^+\pi^0$ must have all of the following properties:

(i) The total Cherenkov energy $E_C$ must be near 940 MeV.
(ii) The event should contain no identified $\mu + e$ decay.
(iii) The neutrino event must have 2 visible, somewhat messy tracks to simulate the showers from $e^+$ and $\pi^0$. These need not arise from $e^+$ and $\pi^0$ directly, but may be from
scattered tracks, multiple tracks with partly overlapping Cherenkov cones, $\pi^0$ production from $\pi^\pm$ charge exchange in the water, etc.

(iv) The energy balance between the tracks should be 35% - 65% or better.

(v) The angle between the tracks should be $>150^\circ$.

The power of the back-to-back angle cut is deserving of a few special comments:

- If the pions from 2-track $\nu$ events were emitted isotropically in solid angle in the lab frame, only 1 in 15 would fall in the solid angle within $30^\circ$ of back-to-back. Note also that the relatively large number of events which pass other cuts but lie outside the back-to-back angle cuts provide a good measure of the expected background inside the cuts.

- The Lorentz boost from the resonance rest frame to the lab frame where the angles are measured tends to depopulate the backwards hemisphere of solid angle. Thus an angular distribution of pions which is isotropic in the resonance rest frame results in a lab angular distribution which is peaked away from back-to-back.

- Pion emission in the resonance rest frame is not quite isotropic. The resonance 'remembers' the direction of production and preferentially emits the pion along the lepton direction, that is, away from back-to-back. This is
however a small effect, and occurs primarily at neutrino energies above 1 GeV [34].

The combined power of these cuts has been more than sufficient to totally attenuate the neutrino background to the \( p + e^+\pi^0 \) decay mode.

III. DETECTOR DESIGN

A. CIVIL ENGINEERING

The detector is located in the Morton-Thiokol salt mine in Fairport Harbor, Ohio at a depth of 2000\' underground. The depth chosen represents a tradeoff between the obvious desirability of as low a cosmic ray background rate as possible, and the rapid rise in excavation costs with increasing depth. A floor plan and side view of the detector are given in Fig. 2.III.A1. The horizontal access tunnel including the utility room and the electronics lab area were excavated from the 20\' thick salt seam at the level in which the main mining operations take place. The main cavity for the detector was mechanically mined from soft shale strata which extend downward for ~60\' below this level.

The utility room contains the electrical power substation, air conditioning, air filtration, water purification, dust control, and fire safety systems.

The water system (Fig. 2.III.A2) is capable of producing
Fig. 2.III.A1. Layout of the underground mine laboratory.
FIG. 2-111. Water purification system for the detector.
80 gallons/minute of water with optical clarity superior to that obtained by triple distillation in quartz. City (drinking) water is piped down the mine shaft, where for safety reasons a pressure regulator is used to reduce the 950 PSI head of water pressure accumulated in the 2000' drop. The water then enters a depth filter containing sand which is used to eliminate large particulate matter. A carbon filter is used to remove dissolved gasses and organic compounds, and a water softener using a conventional ion-exchange resin prepares the water for the reverse osmosis (RO) filter system. Each reverse osmosis cell consists of a polycarbonate plastic semipermeable membrane through which water is forced at 450 PSI of hydrostatic pressure. Only a fraction (~60%) of the water is forced through the membrane; the remainder simply passes through the RO cell and is used to flush the membrane clean and prevent the buildup of excessive osmotic back pressure due to concentration gradients across the membrane. This waste water is then processed in a hierarchic scheme (see Fig. 2.III.A2) to reduce the fraction of waste water to < 5%, stored in a temporary 10,000 gallon waste tank, and pumped back to the surface. The reverse osmosis product water is sterilized by a UV light source which kills any remaining life forms, strained by a final filter which removes their dead bodies, and passed into the detector.
Water already in the detector is continuously refiltered with a two week time constant by either passing it through the entire filtration system, or through a 'fast recirculation' loop (200 gal/min) which simply passes it through a carbon filter and returns it to the detector.

The electronics laboratory is electrically isolated and independently powered from 2 motor-generator sets to provide line power which is free from transients, brief interruptions, etc., which may occur in the AC mains of the mine. The electronics area contains the PMT readout electronics, online computer, magnetic tape drives, etc., as well as a tube assembly, testing, and repair facility.

The detector area is separated from the electronics lab by an opaque wall and contains the water tank and an overhead catwalk (Fig. 2.III.A3(a)). The catwalk is used to gain access to the PM tubes which are suspended in the water. A neutral buoyancy, neutral torque housing design shown in Fig. 2.III.A4 minimizes the external supports necessary to position the PM tubes.

PMT's on the side faces of the detector are suspended in vertical strings of 16 tubes on nylon monofilament lines which are tensioned by lead-filled weights at the ends of each string. A cranking system shown in Fig. 2.III.A3(b) allows these strings to be easily extracted from the water for repair or replacement of individual PMT's. Tubes on the top and bottom faces of the tank are supported on neutrally
Fig. 2.III.A3(a). Corner view of the catwalk area over the detector tank. The PM tubes are suspended in the water volume in neutrally buoyant housings on plastic beams and nylon lines. The surface of the pool is covered by a black Hypalon™ rubber 'floating floor' which minimizes internal reflections in the detector and can support the weight of several people.
Fig. 2.III.A3(b). Side view of catwalk area showing extraction scheme for side PM tubes.
Fig. 2.III.A3(c). Cross sectional view of catwalk area showing plastic beams used to support top (and bottom) PM tubes.
**Fig. 2.11.** PMT housing for 5" hemispherical EMI 9870 PM tube. Housing must withstand two atmospheres of pressure at the bottom of the detector. The PMT housing is neutrally buoyant and exerts no torque when suspended underwater.

Steel screws

Stainless steel cable feed through NYLA-FL0 WATER Tight

Ny whole cable (both arms)

Ballast lead shot

Clamp bolt

Nylon support string MONOFLAMENT NYLON

G30E joint OVER EPOXY TIGHT SEAL RTV WATER

Clear Lucite

End plate

O-ring seal

PM base
buoyant PVC plastic beams as shown in Fig. 2.III.A3(c). These can be raised, lowered, tilted and disassembled from the ends in order to gain access to individual tubes. The mean time to replace a PMT is under 2 hours.

Care is taken to ensure the cleanliness of the water in the tank. The individual tube housings (as well as all underwater components) are constructed from materials carefully tested and chosen so as not to spoil the optical purity of the water, viz: glass, PVC plastic, plexiglass, nylon, stainless steel, RTV cement, high-density polyethylene and Hypalon™ rubber. In particular, many plastics (particularly oil impregnated ones) are unacceptable due to hydrocarbons which leach out over time and interfere with water clarity. All underwater components are baked out (72 hours at 130°F) to remove contaminants. Hypalon™ rubber used for the floating floor (Fig. 2.III.A3(b)) was chemically cleaned to remove contaminants. As a result of these precautions, the total concentration of total dissolved solids in the tank was less than 5 ppm after filling, and has fallen below 1 ppm with recirculation of the tank water through the filtration system.

The liner system for the water tank as shown in Fig. 2.III.A5 consists of (a) a chain link fence supported on ~1m centers by (b) rockbolts extending 4' into the walls, (c) a layer of foamed concrete (specific gravity ~1.1) which we
INNER HDPE LINER (f)
DRAINAGE NET (e)
ROCKBOLT (b)
OUTER HDPE LINER (a)
CHAINLINK MESH (a)
SALT CAVITY
FOAMED CONCRETE (c)

Fig. 2.III.A5. Liner system.
poured between the cavity walls and the outer liner of the
detector as the tank was filled to provide support for the
liner and to fill voids which occur due to irregularities in
the rock wall, (d) an outer liner constructed from 2.5 mm
thick sheets of high-density polyethylene (HDPE), which were
shipped to the mine in rolls and heat-welded into a shape
conforming to the cavity walls, (e) a layer of drainage
fabric to permit water flow between the inner and outer
liners, so that any water that might leak through the inner
liner drains along this fabric to a sump pump, and (f) an
inner liner similar to (d).

The black high-density polyethylene material is
virtually noncontaminating to the water and has an optical
reflectivity of < 1% at normal incidence, which greatly
simplifies event reconstruction software.

B. DATA ACQUISITION SYSTEM OVERVIEW

1. CUSTOM ELECTRONICS

This Sect. outlines the components and capabilities of
the detector electronics. Appendix A contains the details
of my design for the custom electronics which read out the
PM tubes. My main design goal was that the electronics
resolutions in time and pulse height should be substantially
better than those inherent in the PM tubes. In addition,
simplicity, reliability, and low power consumption were
desirable to permit long-term operation underground with a minimum of attention. During the initial run which provided data for this thesis, only 5 single-channel failures occurred in any of the 2048 x 4 channels of TDC, QDC, and programmable discriminators in the custom electronics, and no multi-channel failures occurred.

A System Block Diagram is given in Fig. 2.III.Bl.1. The phototube signals are fed into the custom electronics through a single coaxial cable. This cable brings the high voltage DC down to the tubes and the AC coupled tube pulses up to the electronics. 128 tubes are serviced by a single crate, so that 16 crates are sufficient for 2048 tubes. Each crate contains 16 PC boards each of which contain PMT discriminators and perform the analog functions for time and pulse height measurements for 8 PMT's.

A block diagram of one channel of electronics is given in fig. 2.III.Bl.2. The information recorded for each of the 2048 photomultiplier tubes consists of:

The T1 Time Scale. This records the time between PMT firings and the global detector trigger which provides the time reference for each event. It consists of a TDC (time to digital converter) on each tube which is started by PMT discriminator firings, stopped by the global trigger, and automatically reset after 600 ns if an individual tube firing (due to PMT dark noise) is not followed by a detector trigger. The TDC resolution is 1 ns per count (9 bits),
Fig. 2.III.B1.1. Block diagram of the data acquisition system of the IMB detector.
Fig. 2.III.B1.2. Block diagram of the electronics for a single PMT. 8 identical channels are contained on a single PC card.
substantially better than the best PMT time resolution of 3ns FWHM at high light levels. The full scale range of about 500 ns is comfortably longer than the 150 ns time of flight for an optical photon to cross the cube diagonal of the detector, so that even after allowing for trigger logic propagation delays this time scale will record all of the direct light and virtually all of the scattered light in an event.

The T2 Time Scale records PMT firing times following the global trigger for a period of 7.5 μsec, with 9 bit resolution which corresponds to a least count of 15 nsec. This time scale is used mainly to identify \( \mu + e^+ e^- \) decays following event triggers. Both T1 and T2 can be recorded on the same tube in an event.

Q Scale. This records the pulse height (total charge) of the pulse which caused the first discriminator firing on each tube in an event. It works by sampling the output of an integrating amplifier a short period after the discriminator fires. The QDC (charge to digital converter) resolution is 9 bits with a least count resolution which corresponds to \( \sim 1/10 \) of the average pulse height for a single photoelectron. This resolution was chosen so as to be small compared to the Poisson fluctuations inherent in the collection of single photoelectrons and from electron multiplication in the first dynodes of the tube.
A typical timing sequence for operation of the T1, T2, and Q circuits are given in Fig. 2.III.B.1.3.

**Digitization.** Analog time and pulse height signals on each channel are fed directly into the FET inputs of an operational amplifier which is used as an analog to digital (A/D) comparator. The signals are compared against a reference ramp which is shared among 128 tubes in a single crate. The comparator outputs are scanned digitally over a 3 msec period to determine the time at which the comparator flips and thereby the voltage of the analog signal. This system of digitally scanning the individual comparator outputs is inherently immune to crosstalk and settling time problems of analog multiplexing schemes, and the overhead necessary to digitize each of the 6144 input signals is reduced to 1/4 of a quad op-amp and 1/8 of an 8-input TTL multiplexor. The scanning logic digitizes the data directly into random access memories (RAM's) so that it is immediately available access by the trigger processing hardware.

**Event Buffering.** The software processing of a single detector trigger takes about 120 msec to execute, whereas digitization time is much less. For our trigger rate of 2.7 muons/second, the software generated dead time could be as much as 30% if the detector could not be made live until the event had been read out and processed. To avoid this, the 6144 words in the digitization RAM's are transferred into
Fig. 2.III.B1.3. Typical timing sequence for operation of T1, T2, and Q circuits for 1 channel of electronics. (a) An isolated PM pulse (due to PMT dark noise) initiates the T1 capacitor ramp and stores the pulse height on the Q capacitor. If no detector trigger follows within 600 ns, the T1 and Q capacitors are automatically reset(b). A PM pulse (c) occurring as part of an event which triggers the detector will again initiate a T1 ramp and store the Q information for the pulse. When the global detector trigger signal is returned to the electronics channel (d) it freezes the information on the T1 and Q capacitors so that the T1 capacitor will have a voltage proportional to the time between the PM firing and the detector trigger signal. The detector trigger signal also initiates the T2 capacitor ramp, which continues for 7.5 µsec unless stopped by a second PMT pulse (e).
one of 6 fast multiport buffer memories, and the detector is immediately made live. In this way, dead time is reduced to (digitization time = 2.5 msec) + (event buffering time = 1 msec) = 3.5 msec per event, or ~1% dead time.

The buffered T1,T2, and Q information for each event, as well as such information as the time that the trigger occurred, the water depth in the detector, the electronics room temperature, etc., are monitored and histogrammed by a LSI-11 trigger processor. This information is packed and zero-suppressed by a microprogrammable branch driver (MBD), then transferred to the main memory of a PDP 11/34 computer and written onto magnetic tape. When all triggers are being recorded, one 6250 BPI tape is written every 5 hours.

2. DETECTOR TRIGGERING

The detector trigger was provided by either of two separate systems, both derived from the photomultiplier tube discriminator outputs. Singles rates on individual phototubes (see Fig. 2.III.B2.1) averaged less than 3 kHz, and individual tubes with noise rates above 100 kHz were replaced. The firing rates of various segments of the trigger were consistent with calculations [00] from Poisson statistics, indicating an absence of correlated PMT noise firings in the detector.

The first trigger was logically segmented into 32
Fig. 2.III.82.1. Distribution of dark noise counting rates for PMT's in the detector.

Mean noise rate = 2.8 kHz (up to 20 kHz)
Median noise rate = 2.0 kHz
'patches' or 8x8 arrays of physically contiguous phototubes on the faces of the detector (see Fig. 2.III.B2.2). The discriminator outputs were analog-summed on the backplane of each crate, and the sum was fed into a programmable discriminator so as to be able to select, under computer control, a coincidence level of 1, 2, or 3 tubes within the same patch (Fig. 2.III.B2.3). Each patch is ~8m x 8m across, so that the coincidence resolving time of 55ns was chosen to be slightly larger than the time of flight of a Cherenkov photon across the patch diagonal. In normal operation a 3-tube coincidence level was chosen for each patch, resulting in firing rates for individual patch discriminators typically between 2 and 20 Hz. The final output of this trigger was formed from a two-fold coincidence between patches with a resolving time of 150ns, or slightly longer than time of flight of a Cherenkov photon across the cube diagonal of the detector. The trigger thus required a minimum of 6 PMT's (~25 MeV Cherenkov energy) to fire with a specific topology in the detector. The noise rate for this trigger, after a few minutes of dark-adaption of the PMT array, was typically less than .01 Hz.

A second 'unbiased' trigger required more than a fixed number of tubes to fire within a 55ns window with arbitrary geometry in the detector. The threshold was again set via a software programmable discriminator connected to the analog sum of all discriminator outputs in the system. A
Fig. 2.III.82.2. Trigger segmentation into 32 groups of 64 PMT's organized into 8 x 8 arrays or 'patches' on the surface of the detector.
Fig. 2.11.92.3. Block diagram of the trigger logic. The detector output is generated from either the segmented trigger or the hybrid trigger.
coincidence level of typically 13 tubes provided a negligible rate (less than .01 Hz.) of noise triggers, and an energy threshold of $E_c \sim 50$ MeV.

Since the offline software requirements for reliable reconstruction of events demanded >40 lit PMT's in an event (~160 MeV), the effects of hardware triggering biases were negligible for this thesis.

IV. EVENT SIMULATION OVERVIEW

In analysing an experiment such as this one which is looking for a previously unseen phenomenon, it is important to have an accurate idea of what the phenomenon should look like. The most effective way to do this is to develop a set of event simulation programs which include all known physical processes important in the events, and accurately reflect the statistical and instrumental fluctuations of the detector apparatus.

Detailed parameterizations and formulae used in my event simulation program are contained in Appendix B. The calibration and cross-checks which have been performed between simulated and actual data are described in Sect 2.V. This Section will briefly discuss the assumptions which go into the event simulation code.

The 3 main types of events which are generated are outlined in Fig. 2.IV.1. These are:

(i) Cosmic ray muon events, which are used for
Fig. 2.IV.1. Flow chart for simulation of various event types. All event types were passed through common particle transport and detector simulation routines.
cross-calibration of the event generation programs as well
as simulation and verification of the data reduction
factors. These are generated according to parameterizations
(Appendix B.1) of the muon flux as a function of
(angle, energy) which are based on measurements made at
various depths in mines. The geometrical acceptance of the
detector is simulated. Especially important for these
energetic muons are electromagnetic processes such as
knock-on electrons, direct pair production, etc., which are
simulated both in the water and in the surrounding rock as
described in Appendix B.2. For stopping muons the effects
of nuclear absorption in oxygen, the full V-A (angle, energy)
spectrum of decay electrons [35] and the $\mu^+:\mu^-$ ratio of
1.25:1 of cosmic ray muons [36] are included.

(ii) Neutrino events. Using the $\nu$ fluxes calculated as a
function of angle and energy at our geomagnetic latitude
[33] and the $\nu$ total cross sections [34] it is easy to
calculate the total rate of neutrino interactions. To
convert these rates into expectations for event topologies
and visible energies in our detector it is necessary to
obtain distributions of final state particle types, angles,
and energies in each neutrino energy band. This is done
using Gargamelle [37] bubble chamber events (from a wide
band exposure of freon) which occurred in each neutrino
energy bin. $\nu$ interactions were generated randomly
throughout the detector volume and the identified particle types and energies from corresponding bubble chamber events were fed into the particle transport and detector simulation programs. This procedure cancels out differences between the atmospheric and accelerator neutrino spectra, and produces background estimates which include all of the effects of detector resolutions, particle misidentification, etc. Details of the $\nu$ simulation are given in [00].

(iii) **Nucleon decay events.** For free protons the decay $p \rightarrow e^+\pi^0$ produces exactly back-to-back $e^+$ and $\pi^0$ with 459 and 479 MeV respectively. The $\pi^0$ decays isotropically in its rest frame, the decay products are boosted into the lab frame, and the 3 resulting electromagnetic showers are processed using the EGS programs[38]. For bound nucleons the initial state is chosen to have a distribution of Fermi motion as described in Appendix B.6. The effects of nuclear scattering and absorption of the pions are estimated using an intranuclear cascade model [39] which includes the effects of elastic, inelastic, and charge exchange scattering. The results of our calculation lie intermediate between those of Sparrow[40] and Longemarre [41], and are in good agreement (though slightly more pessimistic) than those of Gabriel and Goodman [25]. See Appendix B.6 for details.

Particles from all 3 event types are processed through a common particle transport and detector simulation program.
outlined in Fig. 2.IV.2. This allows the light yield calibration obtained from cosmic ray muons to be directly related to that expected for neutrinos and proton decay. Electromagnetic showers are followed down to the electron Cherenkov threshold of $T=0.3$ MeV using the EGS code system from SLAC [38] which has been experimentally verified in water [49] as shown in Fig. 2.IV.3. For all charged tracks the additional Cherenkov light from knock-on electrons and direct e-pair production is simulated. Pion absorption, charge exchange, and elastic scattering cross sections have been included (see Chapter IV of Ref. [00]). Particle tracks are followed in segments of 1 cm or less.

Light is generated from each charged track segment according to the Cherenkov formulae for number, wavelength, and angle of emitted photons as a function of $\beta$, thus accurately simulating the folding over of the Cherenkov cone as the particle approaches Cherenkov threshold. Individual photons are followed through the water using light absorption curves and scattering probabilities which have been confirmed by measurements in our detector (Sect. 2.V.D). Shadowing of photons by the tube housings and the geometrical acceptance of the photocathodes are simulated.

The phototube quantum efficiency is simulated as a function of wavelength using the manufacturer's curves, and an overall normalization of the PMT efficiencies (due to photoelectron collection probability and discriminator
Fig. 2.IV.2. Particle Transport and Detector Simulation.

VARIOUS PARTICLE TYPES

- $e^\pm \gamma$ showers
  - EGS code (verified $H_2O$)
- $\pi^\pm$ interactions
  - $H_2O$
- Cosmic Ray $\mu^\pm$
  - (knock-ons, direct e-pairs, etc.)

Charged Track Segments

Cherenkov Photons
- $\cos\theta_c = 1/\beta n$
- $dn/dx d\lambda = 2\pi\alpha\sin^2\theta_c / \lambda^2$

Light Scattering and Attenuation as $f(\lambda)$
(normalized to data)

Shadowing of Tube by PMT Housing

PMT Quantum Efficiency and Collection Efficiency
(normalized to data)

PMT Time Resolution (from data)

PMT Pulse Height Smearing

PMT Dark Noise 2.8 kHz/tube

Detector trigger simulation
Fig. 2.IV.3. Comparison of the EGS electromagnetic shower simulation code with experiments in water [49]. The agreement is good for both the longitudinal and transverse energy deposition.
threshold effects) is adjusted to be 60% to match the data (Sect. 2.V.E). The PMT timing resolution as a function of illumination is simulated to agree with data taken using a 1 ns laser light source in the detector, and agrees well with the timing errors from data events (see Fig. 3.III.7).

The event simulation programs are used to write data tapes in a packed, zero suppressed format identical to that used in data tapes from the mine, and are passed through the entire analysis chain to verify the detection efficiencies.

V. CALIBRATION OF THE DETECTOR

To make meaningful cuts in energy, event containment, and event topology it is necessary to have accurate calibrations of the detector response in pulse height, timing, and phototube efficiency. Timing calibrations are made using programmable light sources. Energy calibrations are discussed in detail in Ref. [00]. This proceeds in two steps: first by linearizing and equalizing the phototube responses using programmable light sources, and then obtaining a scale factor for the absolute energy calibration using the Cherenkov light from straight-through cosmic ray muons. The effects of light attenuation in water, detector geometry, etc., are corrected for on an event-by-event basis to obtain the calibrated energy equivalent light yield ($E_C$) for each event.
Since we must rely on event simulation to reveal the detailed properties of the nucleon decay signature, it is important that the cross-calibration between the simulation programs and the data be checked everywhere that it is possible. This is done by comparing distributions in tube firing efficiencies, pulse height, PMT timing residuals for fits, etc.

A. The Laser/Optical Fiber System for Calibration

The laser test pulse system is the fundamental tool in determining the timing and relative pulse height calibrations of the detector. It is capable of producing short (<\sim1\text{ ns}), approximately isotropic pulses of light which simultaneously illuminate all tubes in the detector at an intensity from \ll 1\text{ photoelectron (PE)} to full scale on the QDC (\sim300\text{ PE}).

Fig. 2.V.A1 shows a block diagram of the laser system. Details of my design for the control circuitry for the laser appear in Appendix A.7. Computer control of a nitrogen laser allows it to be (1) pre-triggered and (2) fired at a programmable time with respect to the computer-generated trigger of the detector. This allows one to determine the detector response over the full range of the T1 and T2 time scales, as well as to perform complete end-to-end checks of the PMT's and detector electronics.

The output of the nitrogen laser is a 337 nm wavelength
FIG. 2.4A1. Block diagram of the laser system.
pulse of between 100 and 500 psec, depending on conditions in the spark gap and lasing head. A small fraction (2%) of the beam is diverted by means of a beam splitter to a fast (<1 ns) UV photodiode which monitors the time and pulse height of the laser output. The remainder of the beam passes through a series of neutral density filters which permit variable attenuation of the beam by more than a factor of $10^5$. Computerized selection of the attenuation occurs via stepping motor control of a pair of wheels, each of which contains a number of filters which can be placed in the beam. A computer controlled mirror then switches the beam between a number of optical fibers which lead to diffusing balls dispersed throughout the detector volume. The width of the light pulse at the end of 40 m of optical fiber was ±1 ns. This is significantly narrower than the ±5 ns time resolution of the phototubes in the detector.

The diffusing ball (Fig. 2.V.A2) consists of a small glass flask filled with DuPont Ludox®, a colloidal suspension of 120 nm silica spheres in a sodium hydroxide solution. Light entering the flask from the end of the optical fiber is rapidly and isotropically dispersed.

Calibration information is normally obtained using a single optical fiber and diffusing ball at a fixed location near the center of the detector. Other diffusing balls are used for light attenuation measurements, tube angular
Fig. 2.V.A2. Isotropic diffusing ball for calibration.
response measurements, consistency checks on reconstruction resolution, etc. One of the optical fibers is connected to a special device used to project a pair of back-to-back rings to simulate two-body nucleon decay (see Sect. 4.III.D).

B. **Timing Calibration of the PMT's**

Data required to obtain timing calibrations of each tube consists of (1) slope and (2) offset information for each of the T1 and T2 time scale TDC's, as well as the information needed to correct for correlations between pulse height and PMT firing time in the data. The slopes and linearities can also be determined by injecting test pulses into the electronics. Since the timing offsets include individual PMT propagation delay times, cable lengths, etc., they can only be determined by the "end-to-end" test provided by the laser light source.

The TDC linearities (<1 ns RMS from best fit line; see Appendix A.2) are such that no quadratic or higher order correction terms are necessary.

The slope coefficients and offsets are obtained by a linear regression of the digitized TDC reading against the laser firing time relative to the detector trigger. Preliminary cuts eliminate tube firings (typically due to dark noise) which fall outside a band 150 ns wide around the position of the expected fit line. Tubes which fail
Chi-squared tests for goodness of fit or whose efficiencies are too low to obtain accurate fits are flagged by the calibration programs and ignored in the analysis. The fraction of tubes failing to calibrate properly was <2% during the initial data taken for this thesis; as of this writing it is ~0.5%.

The timing offsets were corrected for the photon time of flight in water from the laser ball to each tube in order to obtain a calibrated time representing the absolute time that a tube fired in each event. The positions of the tubes and laser ball were surveyed to an accuracy of ±10 cm.

Correlations between phototube pulse height and timing (Fig. 2.V.Bl) arise from the following sources:

(1) Discriminator Walk - changes in the propagation delay of the discriminator as a function of the total charge above threshold. This effect is small compared with effects (2)-(4) below.

(2) Shifts in the time between discriminator threshold crossing and the centroid of the pulse due to the finite rise time (~7ns) of the pulse.

(3) "First Photoelectron Effect" - a statistical effect which occurs because the tube is being operated at the single photoelectron level. If the intensity of an instantaneous pulse of light incident on the tube is such that on average n ≪ 1 photoelectron is produced, one will observe a spread in the time of output pulses referred to as
Fig. 2.V.81. PMT timing distributions at different illumination levels. These are taken from PMT's in the detector and include the effects of light scattering in the water. The timing distribution narrows and shifts to earlier times with increasing light level.
the "single photoelectron timing distribution" (curve (c) in Fig. 2.V.Bl). This spread is due primarily to the range in kinetic energies of the photoelectrons leaving the photocathode and to the electron optics of the tube. For tubes installed underwater in the detector there is a significant contribution to the delayed tail of the timing distribution due to light scattering in the water. For light intensities where \( n > 1 \) photoelectrons are produced, the mean time of arrival is the same but the time of the discriminator firing (which triggers on the first photoelectron output pulse) represents the earliest of \( n \) samplings of the single PE timing distribution and systematically shifts to earlier times as \( n \) increases. A fairly well defined minimum propagation time exists (both in the water and in the phototube multiplying structure) so that this effect will saturate at an illumination level of \( \sim 50 \) PE's after a timing shift (in our tubes) of approximately 15 ns. The width of the timing distribution also becomes narrower, as seen if Fig. 2.V.Bl(a) and (b). This is the largest single source of pulse height time correlations in the data.

(4) Prepulsing. As the light intensity increased to the point where \( n > \sim 200 \) photoelectrons are produced, a significant probability exists for incident photons to pass through the photocathode and produce a photoelectron
directly from the metal surfaces of the first dynode. This produces an output pulse which is smaller than a typical single-PE pulse by a factor of ~4 because of the missing stage of gain in the dynode structure. It is also earlier by 15–20 ns because the light signal has bypassed the transit time of the photoelectron from the photocathode to the first dynode. The reduced pulse height means that single-photoelectron light levels will typically not fire the PMT discriminator. However, at light levels > 200 PE the pileup of several prepulses may fire the discriminator ~20 ns early. This level of illumination occurs only rarely, event for PMT's next to tracks passing through the planes of the tubes as they exit the detector. It does not occur for events contained in the fiducial volume with visible energy < 1GeV.

Each of effects (1)-(4) tends to produce earlier discriminator firings as a function of increasing pulse height. We take advantage of this correlation to improve the effective time resolution of the tubes by applying a pulse height/timing correction of the form:

\[ T' = T + aQ + bQ^2 + cQ^3, \]

where \( T \) and \( T' \) are the raw and corrected PMT firing times, \( Q \) is the digitized QDC reading, and \( a, b, \) and \( c \) are cubic coefficients. The calibration coefficients are chosen
separately for each PMT to minimize the width of the calibrated timing distribution. For additional details see Ref. [00].

C. **Pulse Height Calibration of the PMT's**

Linearizing the pulse height \( Q \) response of the detector was performed via a cubic polynomial fit of the average digitized charge measurement to the laser ball light intensity, as determined from the transmissivities of the neutral density filters in the laser system. These coefficients were then used to compute a linearized \( Q' \):

\[
Q' = aQ^3 + bQ^2 + cQ + d,
\]

where \( a \ldots d \) are calibration coefficients generated independently for each tube. The constant term represents the minimum pulse height required to fire the discriminator of the tube as well as the QDC offset or "pedestal". This linearization is affected by the following considerations:

1. The single largest factor in the nonlinearity of the system was due to the space-charge limiting of the output current pulse from the "venetian blind" dynode structure of the PM tube. This had the effect of reducing the gain of the tube at large pulse height, an effect which mandated the linearization described above. It had the beneficial side effect of increasing the effective dynamic range of the tube by more than a factor of 10 over that obtainable from a linear response curve. This nonlinearity could be reduced
by lowering the operating voltage of the tube, but only at the expense of reducing the gain, efficiency, and time resolution of the tubes.

(2) At low light levels a significant fraction (~10-20%) of the single photoelectron pulses will fall below the discriminator threshold, and the pulse height will therefore not be recorded by the electronics. 'Zeroes' which occur when the discriminator does not fire and no Q is recorded are included into the average Q during the fit. This has the effect of increasing the calibrated weight given to small pulse heights. Qualitatively, the correctness of this procedure is clear: the observed charge from a single PE hit on a tube should be weighted to represent not only the observed charge but also the missing charge from nearby tubes struck below discriminator threshold. Quantitatively, this guarantees that adding the total calibrated Q from a diffuse pattern of tube hits will yield a result which is linear in the light intensity.

Equalizing the calibrated Q response of the tubes requires correcting for 3 effects, all of which enter the Q calibration as simple multiplicative factors:

(a) Differences in the distance from the laser ball to individual PM tubes, which varies between 10 and 20 m. A correction is made for the $1/R^2$ fall-off in the intensity and for the exponential attenuation of laser light in the water.
(b) The angular response of the PM tubes, which was measured underwater using the Cherenkov light from muons. The angular response in the forward hemisphere was fit to good accuracy by \((1 + 0.75\cos\theta)\).

(c) Anisotropy in the radiation pattern of the laser diffusing ball was measured by rotating the diffuser underwater and measuring the calibrated light yield of tubes at various angles with respect to the ball.

After correction for these factors all PMT's had calibrated sensitivities which were linear in light intensity, and equal for equivalent levels of head-on illumination. Details are given in Ref. [00].

**Energy corrections for Vertex Position.** Evaluation of the Cherenkov light yield of an event required that the response of each tube be corrected, using the reconstructed vertex position, for the following effects:

(i) Attenuation in the water of Cherenkov photons. This is given by the convolution of the \(1/\lambda^2\) Cherenkov spectrum, the water attenuation as a function of distance and \(\lambda\), and the quantum efficiency curve of the PMT's. It is therefore more complicated than the simple exponential attenuation at the single wavelength measured using the laser. Attenuation curves for Cherenkov light were determined using the light from fit muons as described in [00]. The assumptions in the event generation programs were based on manufacturer's curves of quantum efficiency
and laboratory spectrophotometer measurements of water attenuation, and were in good agreement with the data as discussed in Sect. 2.V.E.

(ii) The angular response function of the tube. This correction uses the fit form \(1 + 0.75\cos\theta\) and assumes that the event is a point source of light at the fit vertex, which is a good approximation for the short tracks inside the fiducial volume that we are concerned with here.

(iii) Variations in the density of photocathode per unit solid angle about the event vertex. These are compensated for in order to obtain an isotropic sensitivity to Cherenkov light. They arise from 2 sources: (a) The number of phototubes per unit solid angle about the event vertex (which depends as \(\sec^2\theta\) upon the angle of illumination of the wall), and (b) the density of tubes on different faces of the detector (1.04, 1.10, and 0.96 tubes/m² on the north and south, top and bottom, and east and west walls respectively).

The inclusion of these effects result in a detector sensitivity for short tracks which is (a) linear in the Cherenkov light output and (b) independent of track angle and position in the fiducial volume.

D. **Absolute Energy Calibration**

Only one question remains, namely, the absolute energy calibration of the detector. In particular, this involves
the cross-calibration of the event generation programs with the data, for it is only after detailed agreement between the data and simulated cosmic ray muons (whose properties are accurately known) that the detector response can be confidently predicted for simulated nucleon decay events (whose kinematics are assumed known but which unfortunately appear unavailable for detector calibration).

A sample of cosmic ray muons which pass vertically \((\theta < 15^\circ)\) within \(\pm 3\) m of the center of the detector are used for this cross-calibration. The reconstruction of these events is straightforward and the muons were required to have entry and exit points which passed through circles of 3m radius at the centers of the top and bottom faces of the detector. Reconstruction errors for simulated events were 80 cm (entry point), 60 cm (exit point), and 6° (track angle). These errors have a negligible \((<1\%)\) effect on the radiating tracklength since the muons are so nearly vertical. Simulated muons were generated according to the cosmic ray angle and energy distributions (see Appendix B.1) in a range of angles and positions both inside and outside these cuts and passed through identical selection criteria. This guarantees that the systematic effects of the reconstruction procedures would be included identically for both data and simulated events.

The average light level for these vertical muons was 1.9
photoelectrons/firing PMT as compared with 1.2 PE's per PMT for \( p + e^+\pi^0 \) in the fiducial volume of the detector, so that the tubes are calibrated at approximately the light level of interest.

Fig. 2.V.D1(C) is a histogram of the total calibrated Cherenkov light yield from vertical muons from tape 428. The yield has been corrected for track angle deviation from the vertical and for attenuation based on pathlength from the radiating point on the reconstructed track. Also displayed on the Fig. are identical curves for simulated muons of two types: (A) Idealized muons without knock-on electrons, pair production, etc., and (B) Realistically simulated muons with full electromagnetic interactions as detailed in Appendix B.2. For the idealized muons the total radiating tracklength is equal to the muon pathlength through the detector. The 3% width of the energy distribution for idealized muons is essentially what one would expect from the fluctuations in the collection of the ~1000 photoelectrons and represents the inherent energy resolution of the detector at ~4 GeV. The 22% shift in the peak of the energy distribution for realistically simulated muons represents the additional light from the radiating tracks of knock-on electrons, etc., and the the fluctuations in these simulated processes accurately reproduce the shape and width of the actual data. The most probable Cherenkov energy deposition by these vertical muons in the region
Fig. 2.V.D1. Total calibrated light yield for fit vertical muons. (a) Simulated idealized muons without the fluctuations due to knock-on electrons, direct e-pair production, etc. (b) Realistically simulated muons with the above processes included for muons with the energy spectrum at 1600 MVE. (c) Data from vertical muons. The scale factor for the absolute energy calibration is obtained by requiring these curves to overlap.
above the bottom PM tubes is 4.8 Gev. The overall scale of the Cherenkov light yield has been adjusted so that this curve overlaps with the data; an identical adjustment when applied to other simulated events corresponds to the absolute energy calibration of the detector simulation.

E. Tube Firing Efficiencies

The number of tubes firing in an event is most directly related to the calibration of the firing efficiencies of the tubes. This is related to the pulse height calibration, but is a somewhat separate concern. It is possible, for example, to obtain the same calibrated Q with tubes firing with very low efficiencies but high calibrated gains. Since in the analysis certain 'energy cuts' are made by requiring that the number of firing PM tubes lie within specified limits, it is important that the PMT firing efficiencies be accurately represented in the event generation program.

Fig. 2. V. El shows the firing efficiency as a function of the distance from the radiating point on the reconstructed track of muons which passed approximately vertically through the center of the detector. Also plotted are the same curves for simulated muons which are generated by the event simulation program. The different curves which bracket the data represent variations of ±18% in the quantum efficiency x photoelectron capture probability assumed for the tubes. This variation translates directly to the light
Fig. 2.V.E1. Phototube firing efficiencies vs. distance from the radiating point on muon tracks passing vertically through the detector. The upper and lower curves represent variations of ±18% from the nominal values of the simulated efficiency of the PM tubes. The height is sensitive to the overall PMT firing efficiency whereas the shape indicates good agreement with the attenuation curves assumed for Cherenkov light.
yield expected from nucleon decay events. Clearly any variation in the simulated efficiencies exceeding ±10% cannot be allowed from the data. This is a prime consideration in determining the 15% quoted systematic uncertainty in the energy calibration of our detector.

F. Muon Decay Detection Efficiency, Visible Energy, and Muon Lifetime

Decays of stopping muons provide an independent check on the performance and calibration of the detector and the event simulation program. Decay events are selected using a sample of top-entering muons which illuminate ~400 PM tubes and stop near the center of the detector. The selection procedures are detailed in [42]. A similar sample of simulated muons was obtained, and the characteristics of the two samples compared.

**Efficiency.** \( \mu \rightarrow e \) decays are identified by demanding a > 5-fold coincidence of PMT hits in the T2 time scale within a time window of 60 ns. Firing times are corrected for the photon time-of-flight from the reconstructed event vertex to the PM tube. PMT's which fired in the T1 time scale are excluded because of increased probability of PMT afterpulsing due to ion feedback. The efficiency for this procedure is determined by the following considerations:

(i) The electron energy spectrum from \( \mu \rightarrow e \nu \nu \) decay [35]
has an approximately triangular shape which rises linearly to \( \sim 50 \) MeV and cuts off at 53 MeV. Since \( \sim 4 \) MeV corresponds to one observed photoelectron in our detector, \( \sim 20\% \) of muon decays will light less than 5 phototubes.

(ii) \( \mu^- \) which stop in water have an experimentally measured probability [43] of 0.18 of being absorbed before decay. This nuclear absorption rate leads to a reduction of the apparent lifetime of \( \mu^- \) in water from 2.197 to 1.812 \( \mu\text{sec} \). The timing distribution observed for stopping muons in the detector is the sum of two exponentials weighted by the charge ratio \( \mu^+ : \mu^- = 1.25 : 1 \) for cosmic rays.

(iii) The T2 time scale is sensitive for a period starting 250 ns after the PMT hits which formed the global trigger and continuing for 7.5 \( \mu\text{sec} \). Missing very early or very late \( \mu + e \) decays leads to an \( \sim 15\% \) detection inefficiency.

(iv) Even after correcting for the time of flight from the reconstructed vertex of the event tubes are occasionally eliminated by the 60 ns coincidence window due to light scattering or timing fluctuations in the tubes.

(v) Tubes with T2 firings are also eliminated by the requirement that they have no T1 hit (7\% of the tubes would have fired in a typical \( p + e^+ \pi^0 \) event) or if they did not calibrate properly. This causes the muon decay detection efficiency to decrease somewhat with increasing numbers of tubes hit on the T1 time scale.
(vi) The detection efficiency for detecting 2 muon decays will be slightly smaller than the square of the single \( \mu \) decay efficiency due to the possibility of pileup of both \( \mu \) decay signals within the same 60 ns coincidence window.

Each of the above effects has been taken into account in the event simulation program in order to calculate the expected detection efficiency. The sample of simulated stopping cosmic ray muons has a measured detection efficiency of 57\%, whereas the sample from the data gives 58\%. After correction for the effect (v) above, the detection efficiencies for \( \mu + e^+ \nu \) decays in events with <50 PMT's in the main trigger is 65\% for \( \mu^+ \) and 59\% for \( \mu^- \).

Visible Energy. The coincidence multiplicities from identified \( \mu + e^+ \nu \) decays provides a further check on the absolute energy calibration of the detector, since the number of tubes firings expected from \( \mu + e \) decay will depend on assumptions made about light yields, tube efficiencies, light attenuation in water, etc. Agreement between the data and the event generation programs (see Fig. 2.V.Fl) is considerably better than the 15\% systematic error quoted for the absolute energy calibration of the detector, and indicates that the cross-calibration obtained using 4 GeV energy depositions from muons is valid for 50 MeV electromagnetic showers as well.

The contamination in the T2 time scale from accidental late timing coincidences is evaluated using laser triggers
Fig. 2.V.F1. Second timing coincidence multiplicities for stopping muons from data and simulation. This is an independent check on the energy calibration of the detector.
with different numbers of PMT's lit in the T1 time scale (see Fig. 2.V.F2). The probability of accidental coincidences was found to be \(~1.5\%\) for events with less than 100 PMT's illuminated, and decreased with increasing illumination levels due to effect (v) above. All observed accidental coincidences had minimal (5-fold) multiplicities.

**Lifetime.** The effective cosmic ray muon lifetime in water was compared with expectations by measuring the mean time of second timing coincidences. This represents an effective average over the T2 range of sensitivity, since the expected decay time distribution is the sum of two exponentials weighted by the cosmic ray charge ratio of \(\mu^+:\mu^- = 1.25:1\). The mean times of 1.77 \(\mu\)sec for the data and 1.78 \(\mu\)sec for the simulation (see Fig. 2.V.F3) were in good agreement.
Fig. 2.V.F2. Accidental late coincidence probability vs. level of illumination for laser light triggers.
Fig. 2.5.F3. Time distributions of second timing coincidences for stopping muon data, and simulation.
CHAPTER 3 — DATA REDUCTION

"If it's not Howie it's a tube belching".
- Bruce Gilbert Cortez

I. Introduction

The goal of the data reduction procedures is the identification of all events originating in the fiducial volume of the detector and having energies comparable to those expected for nucleon decay.

Two energy ranges are of interest here: firstly, the broad range from $0.2 < E_{\gamma} < 1.8$ GeV (40 to 300 lit PM tubes) in which we identify neutrino interactions; and secondly, the restricted range from 125 to 225 lit PMT's (940 MeV ±30%) in which we search for $p + e^+\pi^0$ events. Recall that the number of lit PM tubes (NPT) expected from $p + e^+\pi^0$ in hydrogen is NPT = 175±17 (statistical)±17 (systematic). The data reduction proceeds in the following steps, for which the rejection factors are indicated in Fig. 3.1.1:

1. An initial energy cut between approximately 0.2 and 1.8 GeV, which is actually made by requiring that the number of tubes firing in an event (NPT) lie in the range 40 < NPT < 300. This NPT cut correlates well with the event energy and can be made simply, quickly, and without calibrations by the computer in the mine.
Fig. 3.I.1. Number of events surviving each stage of analysis. The last stage (event scanning) is only necessary for the ν analysis.
(2) An approximate event vertex determination is made by fitting the timing pattern of the tube hits to an instantaneous point source of light. This approximation will be good for the short tracks in the events which pass the NPT cut. This vertex position is used to make a liberal fiducial volume cut which saves events with fit coordinates more than 1.5 meters inside of the planes of the tubes.

(3) Use of a higher resolution fitter which makes use of the geometry of the Cherenkov radiation to identify the starting point of the single track which lights the largest number of tubes in the event. Events which fit well to a single entering track are rejected, and again a fiducial volume cut is made on the position of the best-fit vertex.

For the $p \rightarrow e^+\pi^0$ analysis this is the last step necessary for the identification of the 0.23 contained events/day which fall in this energy range. The survival efficiency for simulated $p \rightarrow e^+\pi^0$ events from hydrogen is plotted as a function of position in the detector in Fig. 3.I.2, and averages $\sim$90% inside the fiducial volume. The survival efficiency for cosmic ray neutrinos simulated in the fiducial volume is plotted as a function of the number of lit PMT's in Fig. 3.I.3, and averages $\sim$75%.

For the neutrino analysis, hand-scanning on a color
Fig. 3.I.2. Survival efficiency for simulated p → e⁺π⁰ events in hydrogen vs. distance in from the planes of the PMT's. Dotted line: efficiency for surviving the initial energy cut and the point fit. Dashed line: efficiency for surviving the single track fit. All events reaching this stage would be hand-scanned for the neutrino analysis. Solid line: efficiency for being successfully reconstructed as p → e⁺π⁰ candidate.
Fig. 3.I.3. Survival efficiency for cosmic ray neutrinos simulated in the fiducial volume vs. number of lit PM tubes (NPT), through different stages of the analysis. Dotted line: point fit. Dashed line: quick single track fit. Solid line: all program cuts.
graphics event display is performed to make small (typically <1m) adjustments to the fit event vertex position, and a final fiducial volume cut is made 2m in from the planes of the tubes. This additional step in the analysis is necessary because the small numbers of lit PM tubes in the neutrino energy range result in a certain probability of misconvergence of the fitting routines. ~2.5 events/day are scanned, resulting in 0.86 event/day being accepted as having a vertex in the fiducial volume of the detector. This rate is consistent with that expected for neutrino interactions.

In the p + e^+π^0 energy range the data reduction chain is totally automated, and all data cuts are independent of track angle and position within the fiducial volume of the detector. This feature is necessary to allow one to evaluate the contamination of the final event sample from the heavily downward-going background. The fact that the saved event sample (30 events) contains no excess which is either downward-going or near the top and sides of the detector is clear evidence that the muon and muon-associated backgrounds have been successfully attenuated.

The neutrino analysis chain contains the additional hand-scanning stage which could potentially introduce biases in the surviving event sample. Such scanning biases could to some extent mask the presence of entering backgrounds, but the events which are eliminated are generally
unambiguous, and we believe that these effects are small. Again, the surviving sample of 112 events is uniform in position throughout the fiducial volume and isotropic in track direction.

The efficiencies and systematic effects of each of the steps in the data reduction are outlined below.

II. Initial Energy Cut (Number of lit PM Tubes)

Initially very broad energy cuts are imposed on the data in order to observe a band of neutrino energies above and below 940 MeV, as well as to allow a conservative margin for systematic errors in the absolute energy calibration of the detector in the search for $p + e^+\pi^0$. These cuts are actually made by requiring that the number of PM tubes (NPT) firing in the event lies in the range $40 < \text{NPT} < 300$. For $\sim 85\%$ of the data these cuts are made on-line by the computer in the mine. The approximately linear correlation between NPT and Cherenkov equivalent energy ($E_C$) is shown for both the final event sample in Fig. 3.II.1(a), and for simulated cosmic ray neutrinos saved by the analysis chain in Fig. 3.II.1(b). Fig. 3.II.2 displays the distributions in NPT of the raw data triggers, as well as the same distribution for simulated muon events and shows the position of the NPT cuts. The data reduction factor from the NPT cuts for the raw data was 3.03:1 which was in good agreement with the
Fig. 3.II.1(a). Correlation between the number of lit PM tubes (NPT) and the calibrated Cherenkov energy $E_C$ for 112 contained events.
Fig. 3.II.1(b). Plot similar to 3.II.1(a) for 200 simulated cosmic ray neutrino events saved by the analysis chain.
Fig. 3.II.2. Number of lit PM tubes (NPT) for raw data triggers, and simulation. Also indicated is the NPT distribution expected for free $p + e^+\pi^0$ in the fiducial volume of the detector, as well as the region of NPT for the neutrino analysis. The peak at $NPT \sim 600$ is due to top-entering, bottom-exiting muons.
reduction factor of 3.05:1 for simulated data. The small discrepancies between the NPT distributions for the data for NPT<1000 are presumably due to multimuons (which were not simulated), and for NPT<~25 are due to the detector trigger which was not simulated on this plot. The overall agreement, including the position of the peak due to straight-through muons, is excellent.

The lower cutoff in NPT is determined by the increasing difficulty in reliably distinguishing contained events with <40 lit PMT's from the large number of short tracks from top entering, stopping muons. This lower cutoff is comfortably below the minimum light yield expected for p → e⁺π⁰ even when the π⁰ is totally absorbed in the oxygen nucleus, as shown in Fig. 3.II.3. The upper cutoff in NPT is chosen because of (1) constraints on the amount of computer time necessary to analyze the large number of events with many lit PM tubes in this energy band, and (2) the decreasing efficiency of the point fit procedure (Sect. 3-III) as a fast means of rejecting longer entering tracks. The upper cutoff is approximately a factor of two above the rest mass of the proton. This allows the extrapolation of the neutrino energy spectrum downwards into the nucleon decay region as a means of identifying nucleon decay above the neutrino background for modes which do not possess the specific back-to-back decay signature of p → e⁺π⁰.

The effects of the energy cuts on the data are as follows:
Fig. 3.II.3. Number of lit PM tubes (NPT) expected for $p \rightarrow e^+\pi^0$ in the fiducial volume of the detector. Solid line: free $p \rightarrow e^+\pi^0$ in hydrogen. Dashed line: bound $p \rightarrow e^+\pi^0$ in oxygen. The tail at NPT < 120 is due mainly to events where the $\pi^0$ has been totally absorbed in the nucleus.
(a) Single entering muons with very long ( >7 meters) or very short ( < 1 meter) pathlength through the detector are eliminated. The remaining single muons pass through an edge of the detector and typically do not enter the fiducial volume of the detector -- "corner-clippers".

(b) Multiple muons are rejected, primarily by the high limit of the energy cut.

(c) 56% of neutrino events with $300 \text{ MeV} < E_\nu < 10 \text{ GeV}$ simulated in the fiducial volume are lost. This is due to the peak in the neutrino interaction rate at low energies and occurs mainly from the low NPT cut. The effect of this cutoff is more pronounced for $\nu_\mu$ than $\nu_e$, due to the lower Cherenkov light yield for muons than for electrons of the same total energy in the detector (see Sect. 4.II). Fig. 3.II.4 gives the efficiency curve of the NPT cuts as a function of $E_\nu$ for neutrino events simulated in the fiducial volume.

(d) No $p + e^+\pi^0$ events simulated in the fiducial volume were rejected out of a sample of 1000.

III. POINT FIT

The next stage in the analysis determines an approximate event vertex by fitting the timing information to an instantaneous point source of light. Events originating inside a fiducial volume inset 2m from the PM tubes are
Fig. 3.II.4. Efficiency for surviving the NPT cut vs. neutrino energy deposited in the detector for simulated cosmic ray neutrinos. (Energy which leaves the detector as neutrinos from neutral current interactions was not included in the calculation of the neutrino energy for this plot).
saved by a liberal fiducial volume cut which saves events with point fit coordinates more than 1.5m inside of the planes of the tubes. The corner-clipping muons which dominate the data at this stage of the analysis emit most of their light in the active veto region of the detector, so that the best fit point source for these events typically lies near the edge of the detector and outside of this cut.

The fact that events with short tracks will be well fit by the point-source approximation is seen from Fig. 3.III.1, which plots the point fit timing errors for $p + e^+\pi^0$ decays assuming PMT's with zero timing errors. The HWHM is less than 1 ns, with the tails due to shower development and light scattering in the water. In our detector this width will be dominated by the 5.5 ns width of the PMT time resolution. Similar results hold for events with any number of short tracks originating from a common vertex: the Cherenkov photons leave the vertex with velocity 0.75c whereas the radiating particles move away at $\gamma-c$. Hence if the track lengths of the particles are less than a few meters, the time-of-flight differences are small and the Cherenkov wavefront is fit well by a spherically expanding shell. This is shown in Fig. 2.I.2.

The basic procedure of the point fit is as follows. An initial point $(X,Y,Z)$ is chosen inside the volume of the detector. One then calculates the timing residuals $\Delta t_i$ from all the PM tubes which fired in the event:
Fig. 3.III.1. Accuracy of the point-fit model for $p + e^+\pi^0$ events. This histogram indicates the difference between the arrival times of Cherenkov photons at the PMT's and the arrival time of a spherically expanding wave front from an instantaneous point source of light. The tails are due to shower development and light scattering in the water.
\[ \Delta t_i = t_i - (n=1.33/c)\sqrt{(X-X_i)^2 + (Y-Y_i)^2 + (Z-Z_i)^2} - t_0 \]

The point fit residuals are simply the firing times \( t_i \) of each tube after correction for the time of flight of an optical photon from the fit point to the tube coordinates \((X_i, Y_i, Z_i)\), as well as an arbitrary timing offset \( t_0 \) which corresponds to the starting time of the event. They would all be zero for an event with PMT hits coming from a point source of light at position \((X, Y, Z)\) and time \( t_0 \), but in practice will be spread by the time resolution of the PM tubes, light scattering in the water, etc. One then adjusts the four parameters of the fit \((X, Y, Z)\) and \( t_0 \) to minimize the width of the timing distribution. The evaluation of the 'width' of the distribution of timing residuals is performed by convolution with a test function as described in Appendix C.1. This yields a normalized goodness parameter \( G \) which equals 1.0 for a perfect point source and tubes with zero timing errors, and lies in the range from 0.6-0.8 for properly fit events in our detector. Simpler measures of the width such as Chi-squared are less effective due to the presence of PMT noise firings and the delayed tail from scattered light.

The \( \sim 80,000 \) triggers/day that pass the initial energy cut place severe constraints on the amount of computing time available to analyze each event. In particular, a full
floating point fit of each trigger is not feasible with the amount of computing power available (∼1/4 of a Vax 11/780). My solution to this was to develop a set of assembly language and Fortran routines which perform point fit operations approximately 30 times faster than through direct floating-point calculation. This point fitting program takes advantage of the fact that the tube positions lie on the surface of a 3-dimensional lattice of unit cell 92 x 99 x 105 cm. If the loci of the trial fit points are constrained to be on the same 3-dimensional lattice, certain quantities (e.g., the photon time of flight from the fit point to each tube) is only a function of the differences in the integer lattice coordinates and can be placed in lookup tables which greatly increases the speed of the program. Since the fit point is constrained to lie on the approximate 1m lattice during the fit, all fit vertex positions will lie on this lattice. The quantization error due to the lattice approximation of ±50 cm (worst case for each coordinate) is smaller than the intrinsic resolution (σ = 65 cm for p→e⁺π⁰) of the point fit. The lattice point fitting procedures are described fully in Appendix C.1.

The systematic errors of the point fit are best appreciated by referring to Fig. 3.III.2, which indicates the spread in point fit resolution for various simulated event types, including laser light simulations of two-body nucleon decay. Note that the resolution in each coordinate is ~50 cm for reasonably isotropic events such as triggers from the laser light source (a), and simulated p→e⁺π⁰
Fig. 3.III.2. Number of events with point fit vertices at each lattice position for several event types. This plot was generated by running the lattice point fitter on a number of events which had identical geometries but different fluctuations in timing, knock-on electrons, etc. (a) Laser light simulation of two-body nucleon decay. (b) Simulated $p + e^+\pi^0$ in hydrogen. (c),(d),(e) Corner-clipping muon tracks, which deposit most of their light outside of the fiducial volume. Note that the resolution smearing occurs mainly along the track direction. (f) Top entering 1.5 GeV stopping muon. (g) Single track 0.5 GeV $\nu_e$, ~20% of which will be lost due to smearing along the track direction. Solid line indicates edge of water volume. Events lying on or inside the dashed line are saved by the point fit.
events (b). These are strongly constrained by the opposing timing measurements. For single track events such as Fig. 3.III.2(c)-(e), the error perpendicular to the track axis is small, but the position of the point fit along the track direction is poorly constrained due to the lack of opposing timing measurements. The best fit along the track axis is determined mainly by the average position at which light was emitted, as explained in Fig. 3.III.3. Since the high limit of the NPT cut removes long entering tracks, corner clipping tracks such as Fig. 3.III.2(c) and (d) rarely enter the fiducial volume. Thus this error along the track direction is of little consequence for determining that these lie outside the fiducial volume, and one achieves a rejection factor of ~30 against corner clipping tracks. Top entering, stopping muons such as 3-III.2(f) emit a significant fraction of their light in the fiducial volume, and the rejection efficiency is only ~50%. Stopping tracks are therefore a disproportionately large fraction (~20%) of the events which survive the point fit. (g) fully contained $\nu$ events (which are predominantly single tracks) will be saved with an efficiency of ~80%, with the losses primarily due to resolution smearing along the track direction. In contrast isotropic events such as $p + e^+\pi^0$ simulated in the fiducial volume will be saved with an efficiency of 99.5% by the liberal 1.5m fiducial volume cut.
Fig. 3.III.3. Geometrical origin of errors in point fit vertex for single track events. This shows that the the point fit approximation of a spherically expanding wavefront is a good fit to the expanding Cherenkov wavefront from a short single track. Displacing the fit point at right angles to the track direction creates large timing errors, so that the fit is well constrained to lie along the track axis. For displacements along the track axis the fit is less well constrained, and is determined mainly by the average position at which light is emitted from the track.
A typical distribution of point fit vertices from the data is shown in Fig. 3.III.4, which represents a cross sectional side view of the detector. The events inside the fiducial volume are clearly dominated by corner-clipping muon tracks which pass through the fiducial volume, and reconstruct inside the fiducial volume due to resolution smearing along the track.

The distribution in NPT of the events which survive the point fit is, as expected, dominated by corner-clipping events with large NPT which pass through the fiducial volume of the detector. Fig. 3.III.5 gives the NPT distributions for both simulated muons and the actual data, which are in good agreement.

The vertex resolution of the point fit for single track simulated $v_\mu$ events is shown in Fig. 3.III.6.

Fig. 3.III.7 compares the point fit timing residuals between simulated saved $v$'s and those from the 112 contained events in the data. The good agreement between the two is a check on the simulation of the PMT response.

**Point Fit Goodness Cut**

In order to reject multiple muons, a second cut requires that the point fit goodness parameter $G$ be > 0.5. This cut requires that the timing errors calculated from the fit vertex be approximately consistent with the time resolution of the PMT's. This requirement is not at all restrictive in that it is easily passed by simulated $v$ and proton decay events, laser simulation of nucleon decay events, as well as
Fig. 3.III.4. Point fit vertex distributions for events processed by the lattice point fitter. Cross sectional view of detector from the south. Events are fit on a 1m grid, and those which lie on or inside the dashed line are saved. This corresponds to a fiducial volume cut \(\sim 1.5m\) in from the planes of the RMT's. The saved events are mainly corner-clipping and stopping muons similar to 3.III.2(c), (d), (e), and (f). See also Fig. 3.IV.1.
Fig. 3.III.5. Distribution of number of lit PM tubes (NPT) for events saved by the point fit, for data and simulation. Both samples are mainly corner-clipping muons with large NPT.
Fig. 3.III.6. Vertex errors of point fit for single track events. Errors are smaller perpendicular to the track direction as explained in Fig. 3.III.3.
Fig. 3.III.7. Point fit timing residuals for contained events from data and simulation. This indicates that the timing errors of the PM tubes are being accurately represented in the event simulation programs.
the data events finally accepted as having a vertex inside
the fiducial volume. Distributions in G for these events
are shown in Fig. 3.III.8.

The pointfit goodness cut serves to reject the majority
of multiple corner-clipping muons which pass the initial
energy cut. If the tracks of the multiple muons are widely
separated, the timing pattern of the event is grossly
inconsistent with a point source of light anywhere in the
detector. In this case the position of the best point fit
is generally not meaningful; it may lie inside the fiducial
volume of the detector, but the goodness parameter G will be
sufficiently small (G <~0.4) that the event can be rejected.
If the tracks of the multiple muons lie close together, the
event simply looks like a single corner-clipping muon with a
higher than normal light yield, and the event will be
subject to rejection factors similar to those of ordinary
muons.

This cut rejected no p + e^+ν^0 events out of a sample of
1000 and rejected 3 simulated neutrinos in the range
40<NPT<300 out of a sample of 800. Liberalizing this cut
from requiring G>0.5 to G>0.4 did not result in any new
contained events being identified. We therefore conclude
that it has a negligible effect on the efficiency for
finding single or multiple track events originating from a
common vertex in the fiducial volume of the detector.

The point fit goodness cut is also effective at
Fig. 3.III.8. Distributions of the point fit goodness parameter $G$ for various event types. The point fit goodness cut requiring $G > 0.5$ was easily passed for events of interest.
eliminating triggers which are caused by high voltage arcing when a tube housing began to leak and fill up with water. This happened ~10 times during the 130 live days of data taken for this thesis. Over the space of a few minutes, this produces a series of triggers from tubes lighting up nearby the leaking housing. The number of tubes lighting up is generally <50 so that this is not background to p + e^+π^0. Furthermore, since the light dribbles out over a 1-2 μsec period, the timing of the event cannot approximate a nanosecond point source of light from particle physics events, and the point fit goodness parameter for these events was G <~0.4. The point fit goodness cut then stripped these events from the reduced data sample.

The data which survives the pointfit stage of the analysis consists of 2800 events/day which are predominantly single corner-clipping and stopping muons, plus ~1 contained event/day which are saved with efficiencies between 85% and 99%.

IV. SINGLE TRACK FIT

The next stage in the analysis involves the operation of a single track fitting program to identify the starting point of the single strongest track in an event, which is usually the track with the largest number of lit PM tubes. The fitter is based on a modification of the maximum
likelihood method and makes use of the Cherenkov angle condition and the pattern of hit and missed tubes as well as the tube timings to evaluate the goodness of fit. In contrast, the point fit of the previous stage of analysis uses only the timing information and hence is not as efficient at rejecting single entering tracks.

In the $p + e^+\pi^0$ energy range of 125–225 lit PMT's, a fiducial volume cut 1.75m in from the tube planes is the only step necessary to isolate the contained events. The fitting programs have an efficiency of >90% for saving $p + e^+\pi^0$ nucleon decay events simulated in the fiducial volume, and the cuts used to obtain the sample of contained events contain no directional or positional biases to distort the angular distributions or mask the presence of downward-going entering backgrounds.

For the neutrino analysis, a 1.25 m fiducial volume cut on the results of this fit reduces the number of saved events to ~2.5/day, of which 0.86 event/day are eventually saved after hand-scanning.

The details of the operation of my single track fitting program are described in Appendix C.2. It is optimized for fitting short ($<\sim5$m) tracks, and makes a number of approximations valid for these tracks. The basic fitting routine operates by giving it a point in space, which is hypothesized to be near the starting point of a single short
track. The fitting routine then evaluates 60-100 hypotheses as to the direction and length of a single track originating near the hypothesized start point. Each fully specified track hypothesis as to start point, direction, and track length defines a pattern of tube hits on the walls of the detector, assuming idealized β=1 Cherenkov geometry. The single track fit goodness G_l is then evaluated by comparing the actual pattern of PMT firings to the idealized pattern through the use of a maximum likelihood model which takes into account the effects of scattered light, PMT firing efficiencies at low light levels, etc. (Appendix C.2). The single track goodness also contains a factor which includes the timing information of the tubes as explained in the Appendix. The single track goodness parameter lies in the general range 0< G_l<5.5, with values of 3.5-5 being typical for clean corner-clipping muon tracks.

The general approach is to choose a variety of hypothesized starting points and select the one which provides the best fit as the vertex of a single track. The starting point with the the best single track goodness G_l is identified, and a small correction (<1m) is made by moving the vertex position along the fit track direction to maximize G_l.

The fitter is actually used in two different modes during the analysis:

**Mode I: Quick single track fit.** In the first mode, a
quick attempt is made to fit each trigger to a single entering track. The event is immediately rejected if a good fit to an entering track is obtained. Specifically, rejecting an event requires a fit goodness parameter G1>2.5 for a single vertex position less than 0.75 meters in from the planes of the tubes.

This quick attempt takes advantage of the 'entry signature': this is the cluster of 3 to 5 early tubes which fire near the entry point of a charged track, usually due to a combination of direct and scattered light on the hemispherical PMT's (see Fig. 2.II.A2). Hypothesized track start points are the coordinates of the first 8 tubes to fire in an event and the best point fit position in the fiducial volume of the detector. A good entering single track fit is found in all but ~50 of the 2800 events/day processed with the quick single track fit.

Fig. 3.IV.1 illustrates the distribution of fit vertex positions before and after this procedure. Each of the events in the top Fig. represents a corner-clipping or stopping muon which was fit inside the fiducial volume by the point fit. When these events are processed by the single-track fitter which makes use of the Cherenkov angle condition, they have been correctly identified as downward-going tracks outside of the fiducial volume (bottom Fig.).

The events which survive the quick single track fit are
Fig. 3.IV.1. Events rejected by the quick single track fitter. Cross sectional side view of the detector from the east, with tube positions indicated near the edge of the detector. Dashed line represents the fiducial volume of the detector, inset 2m from the planes of the PMT's. Top: point fit vertices for 400 events saved by the point fit. Inspection reveals that these events are predominantly corner-clipping and stopping muons. Bottom: single track fit vertices for the same events, correctly fit outside of the fiducial volume.
found to be of two dominant types, both of which involve single, top entering, stopping muons with small numbers of lit PMT's:

(i) Fits which are not found. These are stopping muons for which a good fit to an entering track is available, but simply was not found by the quick guess. Often these are events for which the 'entry signature' of several early PM firings was weak or not present due to Poisson fluctuations, so that the procedure to obtain a first quick guess did not try an acceptable entry point for the track. These events are eventually rejected when the more exhaustive search (Mode II described below) correctly identifies the entering point and obtains a good entering single track fit.

(ii) Bad fits for single stopping tracks which either scatter badly before going below Cherenkov threshold or have large fluctuations in the timing of the PMT hits. The quick fit generally finds the correct vertex near the edge of the detector; however, the goodness of these events is poor (G1<2.5). These events are retained until the exhaustive search (Mode II below) verifies that this fit, although poor, is better than any attainable inside the fiducial volume of the detector.

Mode II: Exhaustive Single-Track Fit. The second mode of operation of the single track fitter involves an exhaustive search for the best attainable single track fit on an ~1 m 3-dimensional lattice throughout the detector
volume. The search (detailed in Appendix C.2), proceeds in three steps: (a) identification of the 50 lattice positions (on a 1m grid) which have the best point fit timing, then (b) using each of these points as a vertex hypothesis for the single track fitter, and (c) performing a search on a finer (40 cm) grid in the vicinity of the best single track from (b). The event is rejected if at any point a good fit to a single entering track is found. At the end of the fit a fiducial volume cut is performed on the vertex of the best fit track.

For the p + e⁺π⁰ analysis (125-225 lit PM tubes) this fiducial volume cut saved all events with a fit vertex more than 175 cm inside the planes of the PM tubes. This was the last step necessary to isolate the contained events in this energy range.

For the neutrino analysis (40<NPT<300) a more liberal cut was made 125 cm in from the PMT's. This cut has a higher background (~1.5 entering tracks/day), but also a higher efficiency for ν events which have small numbers of PM tubes hit or multiple overlapping Cherenkov cones.

Single Track Fitter Performance. The vertex resolutions of the single track fitter for simulated cosmic ray νμ and νe quasi-elastic events surviving the analysis chain are given in Fig. 3.IV.2. The best resolution is achieved for single track track muon neutrinos, where the mean error is 70 cm and the probability of errors >~1.5 m is small. The
Fig. 3.IV.2. Vertex errors of the single track fitter for single track $\nu_\mu$ and $\nu_e$ events saved by the analysis. The resolution is somewhat worse for $\nu_e$ due to the showering of the track.
low failure rate of the fitter for these clean tracks is the major reason for the high rejection factor that is achieved against the entering stopping muons discussed in the preceding Section. The mean error for single track $v_e$ was 135 cm due to the showering nature of the track. There is a slight systematic tendency for the fit vertex to back away from the true vertex as shown in Fig. 3.IV.3(b). This occurs since the low energy secondaries of a shower to radiate light at an average angle greater than the Cherenkov angle of 41°. Under the hypothesis that an individual event was due to a showering track, an approximate correction for this effect could be made which would reduce the vertex error by ~30%. No correction for this effect is made in this work.

The vertex resolution for simulated cosmic ray $\nu$ events is given in Fig. 3.IV.4. The vertex resolution is somewhat worse due to the predominance of events with small numbers of hit tubes and the presence of multi-track events. The larger errors occur for events containing tracks with small opening angles and therefore overlapping Cherenkov rings. These overlapping rings have an effective diameter slightly larger than the rings from either track alone, which causes the fitter to back away from the true vertex (see Fig. 3.IV.3(c)) in an effort to include the pattern of hit tubes at the Cherenkov angle of 41°. This tendency to back away from the vertex is reduced for wide angle neutrino events as
Fig. 3.4.3. Geometric origin of single track vertex errors.

a) Clean single track is well fit by idealized Cherenkov geometry assumed by the single track fitter. b) Showering single track illuminates a large, diffuse pattern of MMT firings which causes the fitter to back up along the fit track direction in an effort to include all MMT hits inside the fit cone. c) Two-track event with small opening angle also causes fitter to back up along the fit track direction. d) Wide angle events ($\theta > 2\theta_{\text{Cherenkov}}$) typically are fit well since the fitter "latches onto" the best single track in the event.
Fig. 3.IV.4. Vertex resolution of the single track fitter for simulated cosmic ray neutrinos saved by the analysis.
the Cherenkov cones stop overlapping as shown in Fig. 3.IV.3(d).

The vertex resolution of the single track fitter for simulated \( p + e^+\pi^0 \) events saved by the analysis chain is shown in Fig. 3.IV.5. The single track fitter generally attaches itself to the strongest (largest NPT) track in the event. In the case of a badly scattered or absorbed \( \pi^0 \) this will be the \( e^+ \) track. Events which are recognized as multi-track at the scanning stage generally are assigned a vertex using the point fitter, since this fitter makes complete use of the opposing timing measurements available in wide angle \( \nu \) events and nucleon decays; the single track fitter ignores tube hits outside the fit Cherenkov cone.

The angular resolution for simulated single track \( \nu_e \) and \( \nu_\mu \) is plotted in Fig. 3.IV.6. The angular resolution for \( \nu_\mu \) events (mean error =4°) is better than for showering \( \nu_e \) events (mean error =6°). Both angular errors are consistent with our ability to measure the angle between two back-to-back tracks to within ±15° (Sect. 4.III).

V. EVENT SCANNING

Although the \( p + e^+\pi^0 \) search is totally automated, the more liberal requirements used to identify \( \nu \) interactions necessitate the hand scanning of the events surviving the software filters.

In the neutrino energy range of 40-300 lit PMT's, the
Fig. 3.4.5. Vertex errors for single track fit and final point fit for $p + e^+\pi^0$ from hydrogen, simulated in fiducial volume. The point fit takes advantage of the opposing timing measurements available in $p + e^+\pi^0$ events, and $\sigma \sim 65$ cm.
Fig. 3.IV.6. Angular resolution of single track fitter.
2.5 events/day which survive the software filters are scanned by physicists on a color graphics display terminal. The terminal software provides interactive 3-dimensional perspective changes and rotations (Fig. 3.V.1). This gives the ability to examine each event from various directions, look down the reconstructed tracks to view the Cherenkov rings of the events (Fig. 3.V.2), etc. Various spherical and cylindrical projections of the solid angle about the reconstructed event vertex can be displayed. The event can be played back in time sequence with the firing times encoded in the displayed color. Alternatively, the firing times can be corrected for the photon time of flight from the event vertex to each tube (Fig. 3.V.3). This enables one to visually check the position of the fit vertex. The track fitting routines can be called interactively from the event plotter, so that a new starting point can be given and an improved fit obtained.

Previous cuts made on the vertex positions from the single track fit save all events with a vertex >125 cm in from the tubes. These events are scanned, the fit vertex is updated (by hand if necessary), and a 2.0 m fiducial volume cut is made. All efficiency estimates for events surviving the analysis chain and being correctly identified as contained-vertex events are based on this 2.0 m fiducial volume cut.
Fig. 3.1. Color coded perspective view of event 151/35037.
Fig. 3.V.2. Color graphics display perspective view looking down the reconstructed track of a single track event in the fiducial volume. The ring due to the Cherenkov geometry is clearly visible. Tubes displayed in red fired earlier due to the shorter times of flight to these tubes.

Fig. 3.V.3. Color graphics projection of the RMT firings in the solid angle surrounding the reconstructed vertex of the previous event. The RMT colors (firing times) have been corrected for the time-of-flight from the fit vertex to each tube. The absence of any systematic color shifts indicates that the event has been properly fit. Note the absence of any energy deposition in the backward hemisphere of solid angle. The interior ring in each hemisphere indicates the Cherenkov angle of 41°.
This procedure eliminates 1.5 of 2.5 scanned events/day. The rejected events are primarily of 3 types:

(i) Neutrino events in the active veto region. These occur at an approximately equal rate to those in the fiducial volume, but have a low efficiency for being saved by the point fit. The more restrictive 2.0 m fiducial volume cut eliminates a number of events previously saved by the more liberal cuts from previous stages.

(ii) Dimuon events in which both tracks illuminated a similar number of tubes. In these cases the single track fitter is unable to choose between either track and correctly identify the trigger as containing an entering track. The single track fit vertex is in general meaningless for this type of event and the fit goodness was poor (Gl <~ 1.5).

(iii) Top-entering, stopping muons with a small number of tubes illuminated. These events are occasionally fit in the top 1 or 2 meters of the fiducial volume. For the majority of these events the scanner can find a better fit outside the fiducial volume by providing a better starting point to the fitting routines. That some events of this type are rejected even though the best-fit vertex lies in the fiducial volume results in an inefficiency for detecting neutrinos with similar properties, i.e. downward-going ($\theta_{2} < 30^\circ$) single track neutrinos with 40<NPT<75. This is a
small subset of all neutrino interactions, and the scanning biases introduced into the neutrino sample here are small.

In the $p + e^+ \pi^0$ energy range of $125 < \text{NPT} < 225$, the scanning biases are of course zero since no hand-scanning is necessary.

VI. UPWARD-GOING EVENTS

The upward-going $\nu$ induced events provide an independent check on the efficiency of the single-track fitter for isotropic events. All events which the single track fitter fits as upward-going are flagged and unconditionally saved by the analysis chain. These events are not used as part of the data sample unless they also survived the normal analysis requirements. If, however, isotropically occurring events (such as $\nu$ interactions or proton decay) are being mis-fit and erroneously rejected by the single track fitter, 50% of these should (by symmetry) be saved as upward-going. This is true since all of the software cuts used to analyze the data were independent of track angle and symmetric at all boundaries of the fiducial volume.

Of the 400,000 events rejected by the single track fitting programs only 12 events were fit as upward-going. Since this sample should contain 1/2 of all neutrino and nucleon decay events lost by the single track fitter, they provide an independent measurement of its efficiency.
Virtually no cosmic ray background exists beyond a zenith angle $\theta_Z > 60^\circ$ as shown in Fig. 2.II.A2. It is therefore possible to use somewhat more than the upper $2\pi$ of solid angle in the search for isotropic events, resulting in an efficiency $>50\%$. For brevity, however, I will describe the results of scanning only those events with fit track directions going upwards. Scanning reveals that:

-Six of the 12 events are very high angle ($\theta_Z > 60^\circ$) downward-going entering muons which clipped the bottom of the detector. They were fit outside the fiducial volume and with track directions slightly above 90 degrees due to angular errors of the fitter.

-Three of the 12 are upward-going single track neutrino interactions in the veto region of the detector. There are few of these events because of the low efficiency for events in this region to survive the point fit.

-Three of the 12 are good single track neutrino interactions in the fiducial volume of the detector. All three are single track neutrinos whose timing and topology were approximately fit (except for the track angle) as a single entering muon track and hence lost by the single track fitter. This is consistent with the type of events lost from the neutrino simulation. The number of these events expected from the neutrino event simulation is 4±1 events, which is consistent.
From these three events (plus three more downward-going events which, by symmetry, were presumably also missed) the measured efficiency of the single track fitting routines is

\[
\text{Eff.} = \frac{(112 \text{ saved events})}{(3+3 \text{ lost events} + 112 \text{ saved events})} = 94\% \pm 4\%
\]

No wide angle $\nu$ events or events which were close to being a background to $e^+\pi^0$ proton decay were identified by the "upward-going overrides".

VI. WHY WE DIDN'T THROW AWAY THE SIGNAL.

In a null experiment such as this an overriding concern is whether any of the data reduction procedures have inadvertently eliminated the signal. Here I recapitulate some of the key arguments developed in the previous sections which convince us that this is not the case.

(1) The efficiency for saving events simulated in the fiducial volume of the detector is high: 75% for simulated cosmic ray neutrinos which are predominantly single tracks of $<0.5$ GeV energy, and $>90\%$ for $p + e^+\pi^0$.

(2) The rate and characteristics of the identified contained events are consistent with those expected from cosmic ray neutrino interactions (Sect. 4.II).

(3) The initial 'energy' cut of $40<NPT<300$ comfortably contains the $175\pm17$ range of NPT expected for $p + e^+\pi^0$, even taking into account a $50\%$ systematic error in the absolute
energy calibration, an error that we believe is under 15%.

(4) Rejection factors and distributions of events at early stages of the analysis (the NPT cut and the point fit) agree with the expectations from simulation of the cosmic ray muon background.

(5) The point fit is a simple procedure for which the fiducial volume can be measured and fitter convergence checked using controlled light sources, including simulated two-body nucleon decay (Sect. 4.III). The spatial resolution achieved with these light sources agrees well with computer simulations.

(6) The single track fitter correctly fits the vast majority of entering tracks as downward-going. An examination of all upward-going tracks (a sample of only 12 events, but one which should, by symmetry, contain 1/2 of any mis-fit neutrinos or nucleon decays) reveals an additional number of contained events (3 in 130 days) which is consistent with our efficiency estimates.

(7) The hand-scanning reduction factor for the neutrino analysis from 2.5 events/day to 0.9/day is sufficiently small that it should not introduce large biases in the data.

(8) Cross checks of efficiencies between independent analysis chains developed by other collaborators on this experiment confirm the efficiency estimates for wide-angle neutrino events.
CHAPTER 4 - RESULTS

"While it is possible that the four-legged animals that we see along the highway are grossly deformed horses with large udders, in all probability they are cows".
- Dan Sinclair

I. Muon-Induced Background in the Contained Event Sample

To evaluate the possibility of muon-induced background, we inspect histograms and scatter plots to look for excesses or deficits of events in various regions of the space of position, track angle, and visible energy, and with various cuts applied (single or multitrack topologies, observed $\nu + e\nu\nu$ decay, etc.). The vertex distribution of contained events is given in Fig. 4.I.1. Within the statistics available, they are uniformly distributed in position and isotropic in track direction, as is expected for neutrino induced events. The distribution of the events in position and angle are histogrammed in Figs. 4.I.2 and 4.I.3.

If one uses the number of downward-going events minus the number of upward-going events in Fig. 4.I.3 as a measure of the contamination, one calculates

$$ (# \text{ DOWN}) - (# \text{ UP}) = 63 - 59 = 4 \pm 10 \text{ events}. $$

This is consistent with zero and gives a 90% C.L. limit that $< 20\%$ of the events are muon-associated. This limit is in fact stronger if the contained events are neutrino
From the planes of the PM tubes, edge of the fiducial volume, inset Zm detector. Dashed line represents the indicated at the edges of the in the event. PM tube positions are costheta of the best-fit single track. The arrows indicate the direction. Fig. 4.1.1. Vertex distributions of 112 contained events with $N_{\text{P}}T > 300$. EAST VIEW

TOP VIEW
Fig. 4.I.2. Histogram of X, Y, and Z vertex positions for the 112 contained events. The means of all of the distributions are consistent with zero.
Fig. 4.I.3. Histogram of X, Y, and Z direction cosines for 112 contained events. All distributions have means which are consistent with zero, as would not be expected if a significant fraction of the events were induced by the downward-going muon background, or if the detector were systematically miscalibrated in one coordinate.
induced, since the atmospheric neutrino flux has a downward-going excess of 5–7% which should be reflected in the contained event sample.

A number of other statistical estimates that one may obtain for the fraction of muon-induced contamination in the contained event sample are also consistent with zero. The most significant evidence is the absence of any correlation between tracks being near the edge of the fiducial volume and their downward-going direction: this is clearly visible for rejected tracks outside the fiducial volume in Fig. 3.IV.I, but is not in evidence for the events inside the fiducial volume in Fig. 4.I.1.

The vertex distributions for the 30 contained events in the $p + e^+\bar{\nu}^0$ energy range of 125–225 lit PM tubes is displayed in Fig. 4.I.4. Again, within statistics the events are uniform in position and isotropic in track direction. These events were identified by a totally automated set of programs which contained no direction or position dependent biases, so that statistical arguments of the type above are directly applicable. For the neutrino analysis there is the possibility that scanning biases could cancel out the contamination from entering backgrounds to produce a net null result. As discussed in Sect. 3.V, the ~1.5 events per day which were rejected by hand scanning were predominantly clear-cut failures of the fitter to converge properly, so that statistical tests for muon
Range of 125 to 225 MeV

analyzed chain in the "p + e+" energy

saved by the totally autonomous
distribution for 30 contained events

Fig. 4.1.4. Single Track Vertex
associated background contamination are still accurate.

The vertex distribution of the 25 contained events with identified $\mu + e$ decays is plotted in Fig. 4.I.5. Again, there is no indication of muon-associated backgrounds.

A histogram of saved events vs. analyzed live time is given in Fig. 4.I.6. The number of contained events is flat in live time; the fluctuations are consistent with a Poisson distribution with a mean rate of of $0.85 \pm 0.08$ events/day.

II. Comparison with Expectations from Neutrino Interactions

We compare the distributions of the contained events to those obtained from a sample of simulated $\nu$ interactions equivalent to 1.75 years of detector operation. Any departures from those expected from neutrinos might signify nucleon decay. The simulated events include charged and neutral current interactions of muon and electron neutrinos and antineutrinos based on the flux calculations of Gaisser [33]. The details of the simulation are discussed in Ref. [00]. Passing these events through the same analysis programs as the data generates the curves plotted in Fig. 4.II.1, giving the expected and observed distribution in Cherenkov energy $E_C$. For events with identified $\mu + e$ decays, $E_C$ can be corrected upwards by 235 MeV to reflect the extra energy implied for these events (see Sect. 2.I). This results in the curves of Fig. 4.II.2.

The absolute rate and energy distribution of contained
Fig. 4. 1.5. Single track vertex distributions for 25 events (out of 112) which have identified $\mu \rightarrow e$ decays.
Fig. 4.1.6. Number of contained events vs. analyzed livetime. The distribution is, within statistics, flat.
Fig. 4.II.1. Cherenkov equivalent energy ($E_c$) distribution for contained events, and simulation based on the $\nu$ flux calculations of Gaisser. The normalization of the simulation has been adjusted downward by 20% to agree with the data.
Fig. 4.II.2. $E_{\text{min}}$ distributions of contained events, and simulation based on $\nu$ flux calculations of Gaisser. $E_{\text{min}}$ is the Cherenkov equivalent energy after a correction of $+235$ MeV for each identified $\mu + e$ decay.
events is consistent with most, if not all, of the events being due to atmospheric $\nu$ interactions. The estimated uncertainty in the $\nu$ flux varies from less than 15% above 2 GeV to a factor of 2 at 300 MeV, so that a significant fraction of the signal with $E_{\text{min}} < 1$ GeV could also be due to nucleon decay. This question must be resolved by making more restrictive cuts specific to different nucleon decay modes.

The $\mu/e$ ratio of the contained events can also be checked for consistency with expectations for the ratio of $\nu_\mu$ to $\nu_e$ interactions. The fraction of events with an observed $\mu+e$ decay is significantly reduced from the cosmic ray $\nu_\mu:\nu_e$ ratio of $\sim 2:1$. This is due to the much greater sensitivity of our detector to $\nu_e$ than to $\nu_\mu$ for $E_\nu \sim 250$ MeV, where the rest mass of the muon becomes important. Also, our muon decay detection efficiencies are 0.65($\mu^+$) and 0.59($\mu^-$). The observed $\nu_\mu/\nu_e$ fraction is increased by the presence of stopping $\pi^+$'s from high energy $\nu$ interactions. Also, 1.5 of the 112 contained events are expected to have an accidental 5-fold second timing coincidence (see Sect. 2.V.F). The fraction of contained events with an identified $\mu+e$ decay signature is (25/112) = 22% ± 4.5%, whereas the neutrino simulation predicts 33% ± 2%. This ~2.5 $\sigma$ discrepancy may be statistical, may arise from uncertainties in the $\nu$ flux, or could be due to the
difficulty in resolving the ambiguities between \( \pi^+ \) and \( p \) in the bubble chamber events used to simulate the neutrino interactions. The 50 days of analyzed data subsequent to this thesis are in good agreement with the simulation, so that the statistical significance of the discrepancy has decreased.

III. EXTRACTION OF \( p + e^+ \pi^0 \) EVENTS

The contained event sample includes 30 events in the \( p + e^+ \pi^0 \) energy range from 125 to 225 lit PM tubes (\( E_C = 940 \) MeV ± 30%). These are predominantly single track neutrino events and thus not candidates for \( p + e^+ \pi^0 \). To isolate \( p + e^+ \pi^0 \) events from the neutrino background, the two-track, back-to-back, equal energy signature of \( p + e^+ \pi^0 \) must be successfully reconstructed. For example, if one track from a \( p + e^+ \pi^0 \) event near the edge of the fiducial volume shoots directly at the nearest wall, the track will illuminate only 15-20 tubes and the event may be difficult to reconstruct. We evaluate the reconstruction efficiency for three different cases: free proton decay occurring in the hydrogen of our water, proton decay in the oxygen nucleus for which only the effects of Fermi motion are present, and 'full nuclear' proton decays in which pion rescattering and absorption in the oxygen nucleus is simulated.
A. Free $p + e^+\pi^0$ in Hydrogen

Free protons simulated in the fiducial volume of the detector invariably yield a pair of back-to-back rings which are clearly visible when the pattern of tube hits is displayed in the solid angle around the reconstructed event vertex. Fig. 4.III.1 is a color graphics display of the back-to-back rings of a typical simulated free $p + e^+\pi^0$ event. Compare this with Fig. 3.V.3, which gives the same display for a typical data event (apparently a single-track neutrino interaction) from the contained event sample. Only about 5 of the 30 events in the energy range of 125–225 l/r PMT's expected for free $p + e^+\pi^0$ have a significant energy deposition in the backward hemisphere. The topologies of these events (i.e. angles and energy balances between the tracks) are clearly inconsistent with the signature of free $p + e^+\pi^0$. Thus simple inspection of the contained event sample on the color graphics display directly shows that we have no candidates for $p + e^+\pi^0$ from the $2.2 \times 10^{32}$ free protons in the fiducial volume of our detector.

Quantitatively one measures the angles between two-track events and calculates the visible energy associated with each track in an event. Simple cuts on these variables allow the signal from free $p + e^+\pi^0$ in the fiducial volume to be retained with high efficiency (no simulated events lost out of 152 surviving previous cuts), while totally eliminating the neutrino signal.
Fig. 4.III.1. Spherical projections of simulated $p + e^+ x^0$ in hydrogen. Note the equal energy depositions in the forward and backward hemispheres. Compare with Fig. 3.V.3.
Isolating the free $p + e^+\pi^0$ signal proceeds as follows:

(i) The energy is required to lie near 940 MeV by requiring that number of lit PMT's lie in the range $125 < NPT < 225$. This is essentially 100% efficient for saving free $p + e^+\pi^0$ simulated in the fiducial volume (see Fig. 3.II.2). This requirement becomes progressively less efficient as event vertices approach the wall of the detector and the energy deposition lies increasingly outside the fiducial volume. Events failing this requirement typically have significant energy deposition outside of the planes of the PMT's, and hence are unlikely to be reconstructable as $p + e^+\pi^0$ in any case. The cut is wide enough that its efficiency is insensitive to the 15% systematic error in the absolute energy calibration of the detector.

(ii) The events are then passed through the analysis chain consisting of the point fit and the single track fitter. The efficiency for free $p + e^+\pi^0$ events simulated in the fiducial volume to survive and be identified as a contained event is $89\% \pm 4\%$, with the losses occurring for events near the edge of the fiducial volume as shown in Fig. 3.I.2.

(iii) The events are then processed through the 2-track fitting program described in detail in Appendix C.3. Roughly speaking, this fitting program maximizes the number of lit PMT's which fire at or near the Cherenkov angle in each of two cones in the solid angle surrounding the fit event vertex. The two-track fitter makes no attempt to
separately identify the $\pi^0$ and $e^+$ tracks in the event, so that there is a 180-degree ambiguity in the angular resolution for each track. However, for $p + e^+\pi^0$ events surviving to this stage of the analysis the fitter has a mean error to the nearest of the two fit tracks of $5^\circ$ ($e^+$) and $12^\circ$ ($\pi^0$). The mean error in the back-to-back angle is $15^\circ$.

The energy asymmetry between the tracks is defined as

$$E_{\text{asy}} = \frac{(E_1 - E_2)}{(E_1 + E_2)} ,$$

where $E_1$ and $E_2$ are the energies of the more and less energetic tracks respectively. The visible energy associated with each track is defined as the calibrated energy from PMT's which fired within $60^\circ$ of the track direction. Note that $E_{\text{asy}}$ is independent of the absolute energy calibration of the detector. Fig. 4.III.2(a) shows a scatter plot of $E_{\text{asy}}$ vs. fit track angle. The circled region contains all 152 simulated free $p + e^+\pi^0$ events passing previous cuts.

The detector sensitivity to free $p + e^+\pi^0$ is evaluated by simulating free $p + e^+\pi^0$ events in the full volume of the detector. For events generated inside the nominal fiducial volume (inset 2m from the planes of the tubes) the efficiency for events surviving all cuts is (148 events saved/166 generated) $= 89\% \pm 4\%$. In addition, 4 events (out of 234 generated) in the veto region pass all cuts. The 4
Fig. 4.III.2(a). Fit angle between tracks vs. energy sharing between the tracks. Crosses: 153 $p + e^+\pi^0$ events simulated in hydrogen. Diamonds: 10 contained events with $125 < NPT < 225$ and identified $\mu + e$ decay. Squares: 20 contained events $125 < NPT < 225$ without identified $\mu + e$ decay. Events without $\mu + e$ decays which fall inside the dashed line are candidates for $p + e^+\pi^0$. 
Fig. 4.III.2(b). Angle vs. energy sharing for 1.75 years of simulated $\nu$ background.
ANGLE VS. ENERGY SHARING
178 P → e⁺π⁰ IN OXYGEN
FERMI MOTION ONLY

Fig. 4.III.2(c). Angle vs. energy sharing for 178 p + e⁺π⁰ events simulated with the effects of Fermi motion only, and which survive the analysis chain. 10% of the events fall outside the dashed line.
ANGLE VS. ENERGY SHARING
150 p → e⁺π⁰ IN OXYGEN. FULL NUCLEAR EFFECTS.

Fig. 4.III.2(d). Angle vs. energy sharing for 150 p + e⁺π⁰ simulated with full nuclear effects in oxygen. 78% of events passing previous cuts appear in the circled region; however, many events failed to pass preliminary energy cuts, etc., so that the overall detection efficiency for p + e⁺π⁰ in oxygen is 54% (see text).
Fig. 4.III.2(e) Angle vs. energy sharing for laser light simulation of two-body nucleon decay. ~150 lit PM tubes vs. 175 expected for p + e^+π^0. The spread in energy sharing was determined by Poisson fluctuations in the number of photoelectrons collected in each track, and was agreed with that expected for free p + e^+π^0 in 4.III.2(a). The angular resolution was better since the diffuseness of the showers from π^0 decay were not simulated.
saved events were generated less than 50 cm outside of the edge of the fiducial volume, and were reconstructed inside due to resolution smearing. Including these 4 events leads to an efficiency, normalized to the fiducial volume, of

\[(148+4 \text{ saved events})/(166 \text{ generated in F.V.})=92\% \pm 5\% .\]

The 30 contained events from the data which fall in the \( p + e^+\pi^0 \) energy band are passed through the same procedures, with the results also displayed in Fig. 4.III.2(a). Most of the data events have only a single visible track, so that the angle between tracks when fit to a two-track hypothesis is \(< 90^\circ \) and has little meaning.

Only a single event (151/35037) falls within the cuts; it has a clear muon decay signal (a 12-PMT coincidence occurring 1.3 \( \mu \)sec after the main trigger) and hence is not a candidate for \( p + e^+\pi^0 \). The event is a dubious \( p + e^+\pi^0 \) candidate on three other grounds: (1) Although it has nearly the correct number of PMT's firing for free \( p + e^+\pi^0 \), \( E_{\text{min}}^\pi=1.2 \) GeV, which is 2 \( \sigma \) high. (2) The two tracks in the event are both clean (see Fig. 4.III.3 and compare with 4.III.1). They are visibly inconsistent with that expected from the gammas from \( \pi^0 \) decay. (3) The reconstructed angle of 139\(^\circ \) is at the limit allowed by Fermi motion and detector resolution for \( p + e^+\pi^0 \). One could defend a curve drawn on the scatter plot which excludes this event while still saving \( p + e^+\pi^0 \) with \( >90\% \) efficiency. There are therefore no candidates for \( p + e^+\pi^0 \) in our data sample.
Fig. 4.III.3. Spherical projection of event 151/35037.
The neutrino simulation of 1.75 years of data yields the scatter plot of Fig. 4-III-2(b). At our current level of statistics, it again seems plausible that most, if not all, of the contained events in this energy range are drawn from a parent distribution of cosmic ray neutrino interactions.

**Limits on Free p + e⁺π⁰.** With no candidates the lifetime limit (at 90% confidence level) is

\[ \tau/B > (2.2 \times 10^{32}) (130/365) (0.92) (1/2.3) \]

\[ \tau/B(p+e⁺π⁰) > 3.1 \times 10^{31} \text{ years (hydrogen only)}, \]

where the factors are respectively the number of free protons in the fiducial volume, the analyzed livetime in years, the detection efficiency normalized to the fiducial volume, and the statistical factor used to obtain a 90% confidence level limit on the basis of no observed events.

A 1.5% correction to the detection efficiency for \( p + e⁺π⁰ \) events vetoing themselves due to accidental 5-fold second timings was not made, since the only data event rejected on the basis of the second timing had a 12-fold coincidence. The probability of an event vetoing itself with a 12-fold accidental coincidence is negligible.
B. **Bound p + e^+π^0 with Fermi Motion**

The effects of Fermi motion on proton decays occurring in water are evaluated by studying a sample of 400 events simulated in the full water volume of the detector. A fraction (2/10) of these are simulated as free p + e^+π^0 from hydrogen and the remainder come from protons moving with a Fermi momentum distribution for oxygen approximated by a spherical box of radius 228 MeV/c (see Appendix B.6 for details). These events are passed through the identical selection criteria used to identify free p + e^+π^0, with results as follows:

(i) The energy cut requiring the number of hit tubes to lie within the range 125<NPT<225 is satisfied by 99.5% of the events simulated in the fiducial volume. This is expected since the primary effect of Fermi motion is to redistribute the energy between the decay products without changing the total energy in the event.

(ii) The survival efficiency through the analysis chain (the point fit and the single track fit) is 90% ± 4% for events generated in the fiducial volume. This is similar to the 89% ± 4% efficiency for free p + e^+π^0.

(iii) The scatter plot of angle vs. energy asymmetry of Fig. 4.III.2(c) clearly shows the effects of Fermi motion. Of the 178 events in the plot, 18 fall outside the circled region for proton decay. This leads to an overall efficiency of 82% ± 5% for events simulated in the fiducial
volume with Fermi motion only, compared with 89% ± 4% for free \( p + e^+\pi^0 \).

C. \( p + e^+\pi^0 \) with \( \pi^0 \) Interactions in Oxygen

The reconstruction efficiency for \( p + e^+\pi^0 \) decays in oxygen is calculated using events generated with an intranuclear cascade model discussed briefly in Sect. 2.IV and detailed in Appendix B.6 and Ref.[39]. In addition to the Fermi motion of the decaying proton, this model includes the effects of elastic, inelastic, and charge exchange scattering of the \( \pi^0 \) in the oxygen nucleus. Charged pions which escape the oxygen nucleus undergo simulated nuclear interactions in the surrounding water. A sample of 1000 events was simulated in the full volume of the detector and passed through all stages of the analysis.

(i) The requirement that the number of lit PM tubes lie in the range of \( 125 < \text{NPT} < 225 \) saved 74±3% of events simulated in the fiducial volume (see Fig. 3.II.3). The 26% of the events lost represent mainly total nuclear absorption of the \( \pi^0 \), so that only half of the rest mass of the proton is visible in the detector. These events would not have reconstructed as two-body \( p + e^+\pi^0 \) candidates even if we had made no energy requirement.

(ii) For events passing the NPT requirement, the survival through the analysis chain (the point fit and the single
track fit) is 87% ± 2%. This does not differ greatly from the 89% ± 4% efficiency for free $p + e^+\pi^0$, since for these events a strong $e^+$ track was always available to the single track fitter.

The angle vs. energy asymmetry scatter plot is given in Fig. 4.III.2(d), which clearly shows the degradation of the signal due to nuclear scattering. Only 78% of the events which survived the NPT cuts and the analysis chain fall inside the circled region which characterized $p + e^+\pi^0$ from free protons.

The cumulative efficiency for $p + e^+\pi^0$ in oxygen is $(214 \text{ events saved})/(425 \text{ generated}) = 50% \pm 3%$ for events generated in the fiducial volume. An additional 17 events out of 575 generated in the water surrounding the fiducial volume were reconstructed inside (due to resolution smearing) and passed all cuts. This leads to an efficiency (normalized to the fiducial volume) of:

$$\frac{(214+17 \text{ events saved})}{(425 \text{ events generated in the F.V.)}} = 54% \pm 3%.$$  

**Limits on $p + e^+\pi^0$ Decay in Water.** The combined detection efficiency for protons in $H_2O$ is $(8/10) \times 54% + (2/10) \times 92% = 62% \pm 3%$, where the error is the statistical error from the event simulation only. With no candidate events, the 90% confidence limit on the partial lifetime is
\[ \tau/B > (2.0 \times 10^{33})(10/18)(130/365)(0.62)(1/2.3) \]

\[ \tau/B(e^+\pi^0) > 1.1 \times 10^{32} \text{ years (all protons in water)}, \]

where the factors above are respectively the number of nucleons in the fiducial volume of the detector, the fraction of protons in water, the analyzed live time in years, the detection efficiency for \( p + e^+\pi^0 \) in water, and the statistical factor used to obtain a 90\% confidence level on the basis of no observed events.

D. Test of Event Reconstruction Using Two-Track Light Source

A two-track light source was constructed as an independent test of the efficiency of the event reconstruction programs. It consists of a laser diffusing ball (suspended near the center of the tank) which was masked to project a pair of back-to-back cones on the walls of the detector. Both cones illuminate a somewhat diffuse pattern of tubes close to the Cherenkov angle of 41°, so that the pattern of PMT hits resembles, e.g., \( p + e^+\gamma \) rather than the more diffuse pattern expected from \( \pi^0 \) decay in \( p + e^+\pi^0 \). The timing pattern of the emitted light is essentially that of a point source. As described in Sect. 3.III and Fig. 3.III.1, this is an excellent approximation to the timing pattern expected from \( p + e^+\pi^0 \). The light level from the laser was chosen such that a mean number of 150 PMT's are lit in the events, slightly less than
the mean of 175 expected from $p + e^+\pi^0$.

The scatter plot of reconstructed angle vs. energy asymmetry with the light source in a fixed position near the center of the detector is displayed in Fig. 4.III.2(e). The fluctuations in energy asymmetry are essentially equal to those expected from simulated $p + e^+\pi^0$. They are the Poisson fluctuations in the collection of $\sim 100$ PE's per track. The angular resolution (a mean error of $8^\circ$ from back-to-back) is significantly better than that ($15^\circ$) obtained with computer simulations of free $p + e^+\pi^0$. This is due to the absence of angular smearing from the $\pi^0$ as well as the difficulty in reconstructing the angle of events simulated near the edge of the fiducial volume.

No reconstruction failures occurred in any of the 1010 triggers in this hardware simulation, and all events survived as free $p + e^+\pi^0$ candidates. Thus the test confirms our expected efficiency of $>99\%$ for two-track events occurring near the center of the detector.

IV. Implications for Minimal SU(5)

The theoretical upper limit from the calculation of Marciano [10] on the basis of the SU(5) theory is

$$\tau/B(p + e^+\pi^0) = 4.5 \times 10^{29\pm1.7} \text{ years}$$

$$< 2.3 \times 10^{31} \text{ years}.$$
The limits obtained in this thesis for both bound and free 
p + e^+\pi^0 are in conflict with this prediction. For the 
central value predicted for the theory, we should have 
observed ~500 such decays. On the basis of this calculation, 
one could conclude that the minimal SU(5) model is ruled 
out.

Well before the theory was put to a severe experimental 
test, a number of escape routes were prepared. Perhaps the 
simplest of these is to argue that 'minimal' SU(5) is not 
really minimal, in the sense that certain mass predictions 
can be improved by complicating the Higgs sector of the 
model [43]. This could extend the proton lifetime by a 
factor of 10-100. With a sufficiently complicated Higgs 
structure, the proton lifetime could be put forever beyond 
experimental test.

Another representative means of extending the proton 
lifetime in SU(5) involves an arbitrary adjustment of the 
generalized Cabbibo mixing angles so that the dominant 
amplitude is e.g. p + \tau^+\pi^0 (kinematically disallowed) 
instead of p + e^+\pi^0.

Perhaps the only safe conclusion is that the SU(5) 
theory has lost some of its predictive power. Discovery of 
a p + e^+\pi^0 decay rate at a level 10-50 times smaller than 
our experimental limit could be accommodated by a slightly 
complicated version of SU(5), or equally well by some other 
Grand Unified Theory.
V. **Ultimate Limits for this Mode**

**in a Water Cherenkov Detector**

What are the ultimate limits that one can set against the $p + e^+\pi^0$ decay mode in a water Cherenkov detector? The evident possibilities for detector improvement are an increase in fiducial mass and/or an increase in the number of photoelectrons collected per event. Clearly an increase in fiducial mass can only be justified if the neutrino background can be rejected.

With the exception of the single event which we reject on the basis of an observed $\mu^+e$ decay, all of the contained events from the data are well separated from the $p + e^+\pi^0$ signal in Fig. 4.III.2(a). This distribution is consistent with that in Fig. 4.III.2(b) from the neutrino background simulation, and it seems reasonable to expect that this Figure is a fair representation of what our data will look like after about 2 years of detector operation.

On the basis of this, one expects, in two years of running, two events falling inside the circled region: one which contains an identified $\mu^+e$ decay signature (we have presumably already seen this event), and one which contains no identified $\mu^+e$ decay and cannot be rejected as a $p + e^+\pi^0$ candidate.

This is certainly a pessimistic assessment of the background since the cuts used in this thesis to isolate the
p + e⁺π⁰ signal are deliberately wider than necessary, to ensure high efficiency and insensitivity to systematic calibration errors. More restrictive cuts could be made in the following variables:

(1) Energy. This analysis makes the energy cut by simply requiring that the number of lit PM tubes lie within 30% of that expected for p + e⁺π⁰. By requiring that the calibrated energy E_c lie within ± 15% of 940 MeV, one could eliminate ~1/2 of the events which appear on the scatter plot, including one of the two events in the circled region. It is the author's opinion that demonstrating a systematic error of <5-8% in the absolute energy calibration of a large water Cherenkov detector will be difficult. Thus the collection of greater than a factor of four more photoelectrons to reduce the width of the energy distribution may lead to only a marginal improvement in this regard.

(2) Energy balance between the tracks. Since this quantity is independent of the absolute energy calibration of the detector, collection of a large number of photoelectrons will improve the resolution of this cut. This is true for p + e⁺π⁰ in hydrogen up to the point where systematic uncertainties due to light attenuation in the water (which are in general unequal for different tracks in an event) dominate. Fermi motion destroys much
of the power of this cut for decays occurring in oxygen. A background reduction of ~30% seems possible by a more restrictive requirement (perhaps $E_{\text{asy}}<0.3$) in this variable. This would eliminate the second event in the circled region of the plot.

(3) Angle between tracks. Resolution here is limited mainly by the diffuseness of the showers from $\pi^0$ decay. Simply tightening this requirement from $\theta>135^\circ$ to $\theta>150^\circ$ attenuates the neutrino background by more than a factor of two, while retaining high efficiency for $p+e^+\pi^0$ (see Figs. 4.III.2(a) and (d)).

(4) Muon decay detection efficiency. Both events in the circled region in Fig. 4.III.2(b) are $\nu_\mu$ induced, although the $\mu+e$ decay is identified in only one due to our detection efficiency of ~60%. Detection efficiencies for $\mu+e$ of 85% for water Cherenkov detectors with large (~20%) fractional coverage of photocathode are anticipated [44]. An efficiency increase to this level could reduce the remaining background from unidentified $\nu_\mu$ by an additional factor of 3, although it would be ineffective against $\nu_e$ interactions.

(5) Showering topology. This is a difficult observable to quantify. However, it is the author's opinion after examining the contained events from the data and simulations that ~1/2 of wide angle neutrino events can be eliminated as
p + e+π⁰ events on the basis of topology alone. Many events have at least one ring which is so 'clean' that it is unlikely to have arisen from a showering track from either e⁺ or π⁰. This situation improves dramatically with increased numbers of lit PM tubes in an event.

In conclusion, by using slightly more restrictive cuts with overall efficiencies of ~50%, it seems likely that in a 4 live year operation of the detector there will be no neutrino-induced events which cannot be excluded as p + e⁺π⁰. This would correspond to a lifetime limit of ~1 x 10³³ years. Any second generation detector with a substantially larger fiducial mass would also need larger fractional coverage of photocathode to allow tighter cuts in energy balance and event topology, as well as to allow improved μ+e decay detection efficiency.
APPENDIX A - DETAILS OF CUSTOM ELECTRONICS DESIGN

A.1 High Voltage Components

The PMT base circuit diagram is shown in Fig. A.1.1. The tube is operated from positive HV since the photocathode is near the water and must be grounded. Since the 2048 PMT bases must operate for long periods underwater, they contain only passive components. The resistive divider chain which sets the DC potentials on the 11 dynodes draws ~150 μA per tube. This allows 16 PMT's (matched for operating voltage) to be operated from a single 3 ma channel of programmable HV supply. Changes in the DC potentials due to current drawn by the PMT are not a problem, even at this low value of base current, due to the low counting rate in the detector.

The PMT base is connected to the above-water electronics by a single coaxial cable (75 Ω 'subminax' with a customized extra-thick PVC shell). HV (+2500 VDC) is distributed to the base on the central conductor, and AC coupled PMT pulses are returned over the same cable. The base is partially back-terminated for AC coupled pulses through a 91Ω resistor. The circuit design and PC board layout of the base are the author's, as they were for all components of the custom electronics.

The signals from the PMT's are brought out of the water in 256 groups of 8 bonded coaxial cables. They pass into
COMPONENTS:
R1 = 91Ω, 1/4 W
R2 = 1.2 MΩ, 1/2 W
R3 = 3.6 MΩ, 1/2 W
R4 = 0.91 MΩ, 1/2 W
C1 = 0.1 μf, 3 KV
C2 = 470 pf, 1 KV

Fig. A.1.1. PMT base schematic diagram.
the electronics area through a baffle in which the cables make a series of 90° turns which avoid light leaks into the detector area.

Each group of 8 cables is terminated on an 8-channel 'paddle' PC board shown in Fig. A.1.2. This distributes the HV to each channel and capacitively couples the PMT pulse from +2500 V down to the readout electronics, which are ground-referenced. The edge connector on the paddle card allows it to serve as a removable connector plug for the group of 8 PMT cables. Additional details of HV components are given in [00].

A.2 The 8-Channel Discriminator/T1 TDC/T2 TDC/QDC Card

This card performs the analog functions of pulse discrimination, pulse charge measurement (QDC), and pulse time measurement (TDC) on two different time scales for 8 channels of PMT outputs. A simplified block diagram and timing diagram for 1 of 8 channels of electronics on the card are given in Figs. 2.III.Bl.2 and 2.III.Bl.3, and a full schematic for the card is given in Fig. A.2.1. Key features include:

(a) The T1 TDC which measures the firing time of tubes on a 1 ns/count scale is started by the discriminator output of a tube and stopped when a detector trigger signal occurs. The PMT's are firing continuously due to dark noise at a rate of ~2.5 KHz, so that the TDC must automatically reset
Fig. A.1.2. Cable Paddle card schematic diagram.

C1 - 0.01 μF 3KV
R1 - 1.2 MΩ 1W
R2 - 1.2 MΩ 1W (socketed)
R3 - 300Ω .25W
Fig. A.2.1. 8 channel card schematic diagram. The sections inside the dashed line are replicated for each of the 8 channels on the PC card.
itself if it reaches full scale of ~500 ns without a trigger occurring. This ramping time allows the detector trigger coincidence to be formed, buffered, and returned to the electronics to provide a common time reference for the event. The actual implementation of the T1 TDC uses a CD4013 CMOS flip-flop to control the state of the TDC. A capacitor ramp is initiated when the discriminator output clocks the flip-flop to a logic '1' state. Subsequent discriminator pulses are ignored and the ramp continues until either (a) a global detector trigger signal is returned to the card, which shuts off the current source responsible for the ramp and 'freezes' the timing information stored on the capacitor for subsequent digitization, or (b) after ~500 ns the timing ramp reaches the reset threshold of the flip-flop, at which time the TDC rapidly (~100 ns) resets the ramping capacitor in preparation for the next pulse. The voltage on the T1 capacitor is digitized to 9-bit resolution following the end of the T2 time scale.

The dark noise counting rate results in a typical dead time fraction of 0.001 for each tube, so that a 1 GeV event with ~200 lit PMT's will have a chance of approximately 1 in 5 of having even a single tube missed due to PMT dead time.

(b) The T2 TDC is sensitive to discriminator firings for a period of 7.5 usec following a detector trigger, with a least-count resolution of 15 nsec. The circuit uses the
other 1/2 of the CD4013 flip-flop which is asynchronously preset whenever the detector trigger signal is inactive. When the detector trigger signal goes active, this preset is released and the flip-flop becomes sensitive to PMT discriminator pulses which clock the flip-flop to a logic 0. Subsequent discriminator pulses are ignored. The T2 capacitor ramps during the time interval between the the global trigger and the time of the first discriminator firing, which resets the T2 flip-flop. The voltage stored on the T2 ramp capacitor is digitized to 9-bit resolution after the end of the T2 time scale. Both the T2 timing and reference ramps are RC exponentials, resulting in a linear T2 scale.

(c) The QDC Circuit measures the total charge produced by the tube in a time window around the discriminator firing. The actual charge measurement involves sampling the output of an integrating amplifier a short time after the T1 TDC is started by a discriminator pulse. This sampled output voltage is preserved on the Q capacitor. The digitized value of its voltage is recorded if the detector triggers; otherwise the T1 TDC resets the Q capacitor after reaching the end of its scale after ~500 ns.

The analog portion of the QDC circuit consists of: i) the integrating amplifier (response time of ~100 ns) formed using two of the transistors on the CA3086 transistor array
IC, ii) a 3N171 n-channel mosfet sampling gate under the control of of the T1 TDC flip-flop, and iii) the 380 pf Q capacitor to store the voltage for digitization and readout. The technique of sampling the output of the integrating amplifier leads to a window of charge sensitivity which resembles the $\delta$ function response curve of the integrating amplifier, in contrast to the square-box sensitivity of a conventional gated QDC. For the low counting rates and fairly reproducible pulse shapes of our PMT's there is little practical difference between the two methods. This was verified by resistively splitting a PMT pulse into two cables and digitizing the same pulses with both the custom electronics and a commercial QDC.

(d) The discriminator is a 1-shot type device with a time resolution $\sigma_t \sim 1$ ns in tests with a fast ($\sigma_t \sim 0.3$ ns) phototube (Amperex 2041) operated at the single photoelectron level. It has a 12v, 60 ns output for CMOS compatibility. The double pulse resolution is $< 100$ ns which permits a PMT to register pulses on both T1 and T2 time scales in the same event. Discriminator threshold is programmable between 25 and 70 mv by means of a DC control voltage from a voltage follower (1/4 TL064) connected to a multiplexed sample & hold circuit. The sample/hold outputs are multiplexed x 64 using CD4051B CMOS 8-channel analog multiplexer IC's on each card and refreshed by scanning logic from a 64 word RAM and 8-bit DAC on the digitizer PC
card in each crate.

Application of a large negative programming voltage to the discriminator disables it from firing. This feature is used to make individual noise measurements on all PMT's at the beginning and end of each data tape by turning off all but one PM at a time. Performing tests of the trigger system and identifying and shutting off dead or noisy tubes is also facilitated.

(e) A Summing Amplifier buffers the sum of the signals from all discriminators onto the backplane of the crate. The output of the amplifier is a current source to simplify termination of the backplane. The outputs from 8 cards (1/2 crate) are bussed together on the backplane to form the analog sum of all discriminators from an 8 x 8 'patch' of 64 contiguous tubes. The trigger is formed as described in Sect. 2.III.B2.

(f) The Digitization Comparators on each of the T1, T2, and Q signal storage capacitors are used to compare the capacitor voltages with reference voltage ramps. The reference ramps are generated during digitization and shared among 128 channels in a crate. The time at which the comparator 'flips' is determined by the point that the reference ramp crosses the capacitor voltage, and the digitally multiplexed scanning circuit records this time in fast RAM's and thereby obtains the digitized value of the capacitor voltage. The advantages of this scheme include:
- Low parts count, cost, and power dissipation since the overhead required to digitize one voltage consists of 1/4 TL064 FET quad op-amp, and 1/8 74LS251 digital multiplexer.

- Negligible crosstalk between channels since the multiplexing is done digitally and the problems associated with analog multiplexing such as capacitive feedthrough, leakage currents, etc., cannot occur.

- High differential linearity due to the ramped conversion scheme. This is important to obtain histograms with equal bin intervals, and is difficult to achieve with successive approximation conversion methods.

- Digitization results are stored directly in RAM in a format convenient for rapid transfer into the event buffer memory.

A.3 The 128 Channel Digitizer Card (Fig. A.3.1)

Sixteen of the 128-channel digitizer cards exist in the system, one in each custom electronics crate. This card controls the operation of, and digitizes the results from, the 16 8-channel cards in each crate. It communicates over a daisy-chained flat cable bus with the CAMAC modules which buffer the event data and accept commands from the on-line computer. It also regulates the low voltage supplies, buffers the global trigger signals to all cards within the crate, and contains the discriminator refresh logic.
Fig. A.3.1. Simplified block diagram of 128 channel digitizer card.
A timing diagram of the digitization/buffer memory transfer sequence is given in Fig. A.3.2.

(a) The digitization scanning logic contains a 13 MHz counter which is cleared at the start of digitization and counts through 15 bits once during the conversion ramp time of 2.5 msec. The low order 6 bits are presented to the comparator scanning multiplexer which repeatedly samples the state of each of 64 comparators. The low order 6 bits simultaneously address the 64x9 RAM's in which the digitized results are stored. The high order 9 bits count comparator scanning cycles. This means they hold the digital value corresponding to the analog value of the reference ramp. These 9 bits are conditionally written into the addressed RAM cell depending on whether the corresponding comparator has flipped. For example, if the signal voltage on a given capacitor is such that its comparator flips between the 41st and 42nd scanning cycle, the corresponding RAM cell will have been written into on cycles 0 through 41 but not thereafter. This leaves a correctly digitized value of 41 stored in the RAM cell.

(b) Generation of the analog reference ramps for the crate proceeds via normal capacitor charging and FET switching techniques. It occurs under the control of the 'digitize' signal from the flat cable. Separate ramps are provided for T1, T2, and Q, allowing for independent scale changes for each without modification of individual channel
Fig. A.3.2. Time sequence for digitization and event buffering (not to scale).
components. Voltage followers are used to buffer the ramps onto the backplane and to receive the signals at each 8-channel card. Both the reference ramps and the individual TDC's use the same reference voltage, viz. the locally regulated +12v supply. As a result, the system has excellent rejection of power supply variations, and the TDC's continue to operate with only a few percent change in slope with the +12v supply reduced to +4v. The analog ramp voltages, as well as the TDC and QDC capacitors on all channels, are reset at the end of digitization for 1 msec. The reset occurs during the time taken to transfer the event into the buffer memory, so that no additional dead time is incurred.

(c) The logic to refresh the discriminator thresholds consists of a 100 KHz free running 6 bit counter, two 64 byte RAM's which store the threshold data for the crate, and two 8-bit DAC's which convert the data from each RAM into analog levels. The counter outputs are also used to provide address lines and strobes to the CD4051 multiplexers on each 8-channel card to perform the sample and hold operation of the analog signal on the individual storage capacitors. During a computer 'write' operation into the threshold RAM's, refresh operations momentarily cease as the counters are parallel-loaded with the address of the channel to be modified. After that, data is entered into successive RAM
locations over the data bus on the flat cable.

(d) Data transfer from the digitization RAM's in each custom crate to the CAMAC crate containing the trigger processing computer occurs over a pair of daisy-chained, flat cable, bidirectional data buses. The digitizer card in each crate is capable of recognizing its own address on the flat cable, and responding to 200 ns random or sequential read or write cycles. Computer write cycles into the digitizer RAM's are not normally performed except for diagnostic routines which test for faults in the digitizer cards, flat cable connections, or event buffer memory modules.

(e) Low voltage DC power supplies (+12v,+5v,-6v) are locally regulated for distribution throughout the crate by 3 terminal series regulators. This avoids problems which might occur due to voltage drops on the power supply leads from the central DC supply for the custom electronics system.

(f) NIM level global trigger signals are received on the digitizer card, buffered to +12v logic levels and distributed to the 8-channel cards on the backplane of the crate. Two trigger signals are used, one to stop the T1 time scale TDC's and another to activate the T2 time scale. The overlap between these scales can be adjusted by varying the relative cable delays between the NIM inputs.
A.4 Custom Electronics Supervisor/CAMAC Interface Module

This module (Fig. A.4) is one of three which connects the trigger processor CAMAC crate via the flat cable daisy chains to the custom electronics crates. It accepts a 100ns detector trigger strobe and generates timing and control signals on the flat cable bus which control the digitization and readout of an event. It also generates the NIM level 'global trigger' and 'computer busy' signals for trigger rate and dead time monitoring. At the end of digitization it relinquishes control of the flat cable bus to one of the buffer memory modules which reads the event data from the custom electronics crates into RAM. The supervisor module also responds to CAMAC function codes by generating appropriate data and control signals on the flat cable bus to effect read and write operations on the discriminator threshold RAM's and on the digitization RAM's in the custom crates.

The timing sequences for event digitization, readout, and buffering are shown in Fig. A.3.2. Digitization is initiated by a 100ns detector trigger pulse from the NIM trigger electronics. The supervisor responds by promptly (< 3 ns) asserting the 'global trigger' NIM output which is fanned out by NIM fanout units to all crates of custom electronics. This signal freezes the T1 and Q information on each channel and activates the T2 time scale.
Digitization begins 15 µsec after the global trigger. This delay allows for completion of the 7.5 µsec T2 time scale. The supervisor asserts the digitize signal on the flat cable bus which initiates the T1, T2, and Q conversion ramps. It generates control signals which clear the digitization counters in all crates. It then emits a 13 MHz 'digitization clock' which continues for $2^{15}$ pulses. At the end of digitization (a) the 'global trigger' signal is deactivated, which resets the TDC's and QDC's of the custom electronics, and (b) the digitize signal on the flat cable is deactivated, which terminates the reference ramps in the custom crates and signals to the buffer memory modules that an event is ready for transfer.

A 'computer busy' NIM output is activated by the supervisor at the time of the global trigger. It is reset by the trigger processor when the event data has been transferred into a buffer memory module, and a different module has been enabled to receive the next trigger. This 'computer busy' signal is used to veto detector triggers, and to monitor the live time of the detector.

For diagnostic purposes, detector triggers, event digitization, and/or event transfers to the buffer memories can be initiated under software control.

A.5 Event Buffer Memory Modules

At the end of digitization, the data in the digitization
RAM's in each of the custom crates is transferred to an event buffer memory so that the detector may be immediately turned back on. The buffer memories are dual-ported devices which are connected both to the flat cable daisy-chain from the custom crates and to the CAMAC crate containing the trigger processing computer. Each buffer module is a dual-width CAMAC unit which contains two independent 6144 x 12 RAM arrays each capable of storing the T1, T2, and Q information for an entire event. They contain logic which recognizes the end of digitization. If enabled by the trigger processor, a buffer memory module seizes control of the flat cable bus from the supervisor module, and emits the appropriate address, control, and 'transfer clock' signals to effect a DMA transfer from the custom crates into its RAM buffer. Both flat cable daisy chains are operated in parallel, resulting in a transfer rate of 6M words/sec and an event buffering time of 1 msec. At any point in time only one of the six event buffers are enabled to receive the next event. All others are available for random or sequential access by the trigger processor via the CAMAC backplane.

A.6 Interaction with Trigger Processor

The trigger processor is interrupted at the start of digitization of each event. The interrupt routine reads out CAMAC modules which contain event-related information such
as event time, live time and dead time scalers, etc. It
then waits until the end of the transfer of the event data
into the buffer memory, enables a different event buffer to
receive the next trigger, and then enables the detector
trigger by resetting the 'computer busy' output of the
supervisor module. The processor then returns from the
interrupt routine and continues processing of the full event
buffers on a first-in-first-out basis. If all six event
buffers become full as a result of a large fluctuation in
the trigger rate, the detector remains dead until the
current buffer processing is completed. In actual operation
this computer induced dead time is small compared with the
1% dead time inherent in the electronics.

On-line event buffer processing relevant to this thesis
consists simply of (1) counting the number of tubes
(Tl hits) in each event for the preliminary energy cut of
Sect. 3.II, and (2) zero-suppressing and packing the event
data for writing onto magnetic tape. Details of the on-line
software and live time monitoring are given in Ref. [00].

A.7 Computer Control of the Laser Calibration System

The components in the laser calibration system are
described in Sect. 2.V.A. Firing the nitrogen laser at a
controlled time requires two signals: (i) a 'precharge'
pulse occurring 40 µsec before the discharge which activates
the high voltage (15 kv) across the lasing head, and (ii) a
firing pulse which switches a Marx generator across the spark gap inside the laser, initiating the discharge in the lasing head. The time jitter between the laser firing pulse (ii) and the light output is 2–3 ns; this is monitored and recorded with each calibration trigger by means of a discriminator and a TDC on the output of a fast photodiode viewing the laser output. The photodiode pulse height is recorded by a QDC gated with the photodiode discriminator output to sense fluctuations in the laser output.

A custom 'laser controller' CAMAC module generates three output signals which provide independent control of the precharge and fire pulses as well as generating a detector trigger at a programmable time with respect to the laser output. This is necessary for the calibration of the T1 and T2 time scales. It uses a 50.000 MHz crystal which is the time reference for the experiment, and an analog interpolation circuit provides resolution in 100 psec steps between oscillator cycles. A fourth output provides a simulated PMT pulse with 12-bit pulse height resolution and programmable arrival time which is used to test electronics channels.

Control of the stepping motors for the laser beam attenuators and the optical fiber switching mirror is provided by a 4-channel CAMAC stepping motor controller and a NIM module containing the power FET coil drivers, etc. Potentiometer feedback of the stepping motor positions is
provided via a CAMAC scanning ADC. This ADC also monitors the water level in the detector, temperature in the electronics room, sump pump operation, DC power supplies, etc.

Different types of laser-induced detector triggers can be selected under software control:

1. Normal (PMT coincidence) triggers can be stimulated asynchronously by laser light.

2. The photodiode discriminator output can trigger the detector, and thus provide events with a fixed time delay between light deposition and global triggers. This is useful for studies of time resolution and electronics drift.

3. Programmable timed outputs of the laser controller module can trigger the detector. This mode is used to generate calibration ramps for the T1 and T2 time scales.

We conclude this appendix on the custom electronics with a table of its specifications (Fig. A.7).

APPENDIX B - Details of Event Simulation

B.1 Generation of the Cosmic Ray Muon Angle and Energy Spectrum

The muon intensity as a function of depth and slant height has been measured in a large number of mine experiments at varying depths [27]. These measurements, together with theoretical and experimental expressions for
**Fig. A.7.  CUSTOM ELECTRONICS SPECIFICATIONS**

<table>
<thead>
<tr>
<th><strong>Number of Phototube Channels</strong></th>
<th>2048</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Channels/Card</td>
<td>8</td>
</tr>
<tr>
<td>Number of Cards/Crate</td>
<td>16</td>
</tr>
<tr>
<td>Number of Crates/System</td>
<td>16</td>
</tr>
</tbody>
</table>

**System Dead Time**

- Digitization: 2.5 msec
- Event transfer to buffer memory: 1 msec
  - Total: 3.5 msec

**Dead Time Fraction at 2.7 Hz trigs.** < 1%

**T1 TDC (fine time scale)**

- Range (before global trigger): 0-511 ns
- Least count resolution: 1 ns
- Linearity (RMS deviation from best fit line): < 1 ns
- RMS jitter: < 0.5 ns
- Measured Drift (1 hour): < 1 ns
  - (1 week): < 2 ns
- Automatic Reset Time: < 100 ns
  - (~550 ns after TDC started by PMT discriminator)

**T2 TDC (second timing scale)**

- Range (following global trigger): 0-7.5 usec
- Least count resolution: 15 ns
- Linearity (RMS from best line): < 1 count
- RMS jitter: < 1 count
- Measured Drift (1 hour): < 1 count
  - (1 week): < 2 counts

**QDC (measures charge on 1st pulse in T1 or T2)**

- Resolution: 512 counts
  - ~0.1 PE/count
- Measured Pedestal Drift (1 week): < 2 counts
**Fig. A.7.** (continued)

**Discriminator**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Programmable Threshold min</td>
<td>20 mV</td>
</tr>
<tr>
<td></td>
<td>max 75 mV</td>
</tr>
<tr>
<td>Threshold Programming Resolution</td>
<td>8 Bits</td>
</tr>
<tr>
<td>Time Resolution</td>
<td>+/- 1 ns</td>
</tr>
<tr>
<td>Discriminator Shutdown</td>
<td>Programmable</td>
</tr>
</tbody>
</table>

**Power Consumption**

- each channel: 0.08 watts
- system: ~250 watts

**Event Buffer Memory System**

- Events Stored/Module: 2
- Number of Modules: 3
- Total Events Stored: 6

- Storage Per event:
  - word size: 6144 words
  - 9 bits

- Data Transfer Rate: 6,000,000 words/sec
  - data transferred on 2 flat cable busses
  - simultaneous event processing & transfer

- Event Transfer Time: 1 msec

**Laser Controller/ Test Pulse Injector System**

- Number of independent outputs: 3
  - laser precharge signal
  - laser firing signal
  - detector trigger

- Crystal Controlled Timebase: 50,000 MHz

- Time Resolution of
  - Analog Interpolation Circuit: 100 psec

- Simulated PMT Pulse Output
  - Programmable Pulse Height
    - (used for test pulse injection into custom electronics): 12 bits
the energy loss of energetic muons, enable the unfolding of
the dependence of the cosmic ray muon intensity as a
function of energy, angle, and overburden. This is plotted
at our depth in Fig. 2.II.A3. We use the parameterization
due to Menon and Murthy [45]

\[ I(h, \theta, E_\mu) = I_V(h \sec \theta) \times F(\cos \theta, E_\mu), \quad \text{where} \]

\[ I_V(h) = \frac{164}{(h+400)} \, h^{-1.53} \exp(-0.00065h) \]
\[ = \text{vertical muon flux (cm}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1}) \]

\[ F(\cos \theta, E_\mu) = \frac{90 + E_\mu}{(90 + E_\mu \cos \theta)} \]
\[ = \text{Pion decay probability} \]
\[ \quad \text{enhancement factor.} \]

\[ E_\mu = \text{muon energy in GeV}. \]
\[ h = \text{depth in meters of water equivalent (MWE)} \]
\[ \theta = \text{muon angle as measured from the vertical} \]
\[ \quad \text{(zenith angle)} \]

Muons at the detector are generated by the following steps:

1. A random position is chosen across a 'roof' at the
   water level of the detector. The size of the roof
   (80m x 80m) is about 4 times the linear dimensions of the
detector. This is sufficiently large that this procedure
introduces a negligible bias for muon angles <60° where the
cosmic ray muon intensity cuts off due to the large slant
height.
(2) A random track angle is chosen (in the downward-going hemisphere), and an energy is chosen with a flat distribution out to 10 Tev. The generation of the track angle includes a factor which corrects for the \( \cos \theta \) acceptance of the flat roof.

(3) The track is rejected or accepted according to the muon intensity calculated from the formulae above.

(4) The event is rejected if the track does not pass through the detector volume.

(5) Muon event simulation begins 5 m of track length (> 10 radiation lengths) before the muon enters the tank. The low energy \( \gamma \)'s and electromagnetic shower fragments which accompany the muon are generated in the surrounding rock and enter alongside.

B.2 Electromagnetic Interactions of Muons

To use the Cherenkov light yield of cosmic ray muons as an absolute energy calibration, the detector's response relativistic charged secondaries resulting from the electromagnetic interactions of the muons must be accurately evaluated. These induce a shift of 22% in the most probable deposition of Cherenkov light for muons with the energy spectrum encountered in our detector (see Sect. 2.V.D). A much larger shift in the mean of the distribution occurs. Moreover, the showers contribute substantially to the fluctuations in both the energy and topology of cosmic
ray muons. They must be accurately represented to obtain a meaningful comparison of the reduction factors, etc., for the simulation and data. In order of importance, these are:

1. Knock-on (delta-ray) electrons, atomic orbital electrons which have suffered a violent electromagnetic collision with the muon.

2. Direct positron-electron pair production in the field of the nucleus.

3. Muon Bremsstrahlung.

4. Photonuclear (deep inelastic) processes in which a nucleus interacts with a virtual photon from the muon.

Energy transfers ($E' > 2$ GeV) are not of interest either as background to nucleon decay or as a factor in the position of the $\sim 4$ GeV calibration peak from straight-through muons. For energy transfers less than this, only the significant processes (1) and (2) (see Fig. B.2.1) are included into the event simulation.

1. Knock-on electrons, which contribute $\sim 90\%$ of the shift of the calibration peak. The production probability for electrons of energy $E'$ per cm of track of a muon with energy $E$ is given by [46]:

$$\phi(E,E') = 0.30 \frac{Z}{A} \frac{m_e c^2}{\beta^2} \frac{dE'}{(E')^2} \left[ 1 - \frac{\beta^2}{E_{\text{max}}} ^2 + \frac{E'_{\text{max}}^2}{2E + Zmc^2} \right]$$

where $Z$ and $A$ are the atomic number and weight of the medium, $m_e$ and $m$ are the masses of the electron and muon,
Fig. B.2.1. Probabilities for electromagnetic energy transfers from cosmic ray muons at 1600 MWE.
βc is the velocity of the muon, and $E'_\text{max}$ is the maximum kinematically allowed energy transfer,

$$E'_\text{max} = 2m_e c^2 \frac{p^2 c^2}{m_e^2 c^4 + m^2 c^4 + 2m_e c^2 \sqrt{p^2 c^2 + m^2 c^4}}$$

The dominant features are the $1/e^2$ falloff of the production with increasing knock-on electron energy, and the weak dependence on muon energy (see Fig. B.2.2)). The correction term [in brackets] suppresses interactions which transfer a significant fraction of the muon energy. It is essentially unity for the vast majority of knock-on electrons in our detector.

The kinematics of the knock-on process are such that at asymptotically high muon energies the angle between the electron and the parent track is a single-valued function of the energy transferred to the electron. Although the angular divergence of the knock-ons is broadened by subsequent multiple scattering and shower development, the directional information from the Cherenkov light from delta-rays is substantially correlated with the parent track direction.

A lower cutoff of 0.5 MeV is imposed on the energy of generated knock-on electrons for reasons of computational speed. The error in the light yield due to this cutoff is negligible. After calculation of the electron and muon scattering angles, the EGS routines (see Sect. B.5) are called to simulate full shower development of the electron.
Fig. B.2.2. Pair production and knock-on cross sections as a function of muon energy $E_\mu$. 
(2) Pair Production. Cross sections for direct electron pair production are taken from Koukoulis and Petrukhin [47]:

\[ \sigma(E, \nu, \rho) = \frac{2}{3\pi} (Zar_0)^2 \frac{1-\nu}{\nu} \left[ \phi_e + \frac{m_e^2}{m_\mu^2} \phi_\mu \right] d\nu d\rho, \]

where

\[ \phi_e = L_e \left( \frac{(2+\rho^2)(1+\delta) + \xi(3+\rho^2)}{\ln(1+1/\xi)(1+\rho^2) + \frac{1-\rho^2-\delta-3\rho^2}{1+\xi}} \right), \]

\[ \phi_\mu = L_\mu \left( \frac{(1-\rho^2)(1+3\beta/2) - (1+2\beta)(1+\rho^2)/\xi}{\ln(1+1/\xi) + \frac{1-\rho^2-\beta + (1+2\beta)(1-\rho^2)}{1+\xi}} \right), \]

\[ L_e = \ln \left( \frac{KZ^{-1/3} \sqrt{1+\xi}(1+Y_e)}{1+2m_eK\sqrt{e} Z^{-1/3}(1+\xi)(1+Y_e) \text{Ev}(1-\rho^2)} \right), \]

\[ L_\mu = \ln \left( \frac{(Km_\mu/m_e)Z^{-1/3} \sqrt{1+1/\xi}(1+Y_\mu)}{1+2m_eK\sqrt{e} Z^{-1/3}(1+\xi)(1+Y_\mu) \text{Ev}(1-\rho^2)} \right), \]

\[ Y_e = \frac{5 - \rho^2 + 4\delta(1+\rho^2)}{2(1+3\beta)\ln(3+1/\xi) - \rho^2 - 2\delta(2-\rho^2)}, \]

\[ Y_\mu = \frac{4 + 3\rho^2 + 3\delta(1+\rho^2)}{(1+\rho^2)(3/2 + 2\beta)\ln(3+\xi) + 1-3\rho^2/2}, \]

\[ \beta = \frac{\nu^2}{2(1-\nu)}, \quad \nu = \epsilon/E, \quad E = \text{muon energy}, \]

\[ \xi = \frac{m_\mu^2 \nu^2 (1-\rho^2)}{4m_e^2 (1-\nu)}, \quad \epsilon = \epsilon_+ + \epsilon_- = \text{total pair energy}, \]

\[ \rho = \frac{\epsilon_+ - \epsilon_-}{\epsilon} = \text{e-pair asymmetry coefficient}, \]

\[ r_0 = \text{classical electron radius}, \quad \alpha = 1/137, \]

and \( K = 190. \)
For muons of our energies, both the positron and electron are emitted essentially along the parent track direction. I have therefore numerically integrated the cross section over the e-pair asymmetry parameter $\rho$. To permit rapid generation of events, I have integrated as well over the total e-pair energy $E'$ (= $\epsilon$ in the above formulae) to obtain a total cross section with an approximate $\ln^2(E_u)$ dependence (Fig. B.2.3). Pair production events are generated using this total cross section, and the actual energy transfer $E'$ is generated from the differential cross section $\sigma(E_u,E')dE'$ only when a pair production event has taken place.

The rapid rise in the pair production cross sections with muon energies (Fig. B.2.3) means that for sufficiently high $E_u$ the energy transfers $E'$ are dominated by pair production at all but the lowest $E'$, where knock-on electrons dominate. In particular, for the energy transfers of 0.5 to 1 GeV, which are important in determining the width of the 4.8 GeV calibration peak for straight-through muons (see Fig. 2.V.D1), this crossover between knock-on and pair production dominance takes place at $E_u \sim 300$ GeV. This is close to the mean energy of muons in the detector ($\sim 200$ Gev; see Fig.2.II.A3). Thus pair production and knock-ons have similar contributions to the width of the calibration
peak. However, the $1/E^2$ low energy divergence of the knock-on cross section, when integrated down to the Cherenkov threshold cutoff, creates a substantially larger shift in the peak.

B.3 Cherenkov Light Generation and Propagation

The number of Cherenkov photons radiated with wavelength between $\lambda$ and $\lambda+d\lambda$ by a charged track segment of length $dx$ is

$$\frac{dN}{dx d\lambda} = \frac{2\pi a \sin^2 \theta_c}{\lambda^2}$$

$a = 1/137$,

$\theta_c$ the Cherenkov angle given by

$$\cos \theta_c = 1/\beta n,$$

and

$\beta c$ is the velocity of the charged particle.

The index of refraction in water is $n=1.33$ and varies less than 1% over the spectral range of sensitivity of the phototubes. This variation is neglected both in the calculation of the Cherenkov angle and the velocity of propagation of the optical photons. For $\beta < 1/n \sim 0.75$ in water the Cherenkov angle becomes imaginary and the light output ceases.
Cherenkov photons are generated according to the above formulae at the Cherenkov angle appropriate for each charged track segment and at a random azimuth about the start point of each 1 cm segment. For reasons of computational speed it is desirable that the number of radiated photons per unit wavelength interval be folded immediately with the quantum efficiency and the collection efficiency (taken to be 0.60) of the phototube. The quantum efficiency curves represent the manufacturer's nominal values [48]. Individual tubes exhibited variations of ± 20% in efficiencies and these variations are not simulated. The collection efficiency is the probability that an electron leaving the photocathode will result in a PMT pulse above discriminator threshold. It is not observable independently from the quantum efficiency in our tubes, and can be viewed as an overall normalization factor in the average PMT efficiency. It was adjusted to match the PMT firing efficiencies of the data (see Sect. 2.V.E).

Light attenuation is simulated as a function of wavelength using the curve given in Ref. [00]. This curve represents the losses due to both attenuation and large angle scattering. An individual photon is generated at a specific wavelength and given a maximum length from an exponential distribution based on this curve. If the photon flight path exceeds this length before striking a
PMT photocathode or a wall, it is considered to have interacted (i.e. either been absorbed or scattered).

Isotropic light scattering is simulated for photons in the region above 400 nm by randomly choosing the direction cosines and re-emitting a fraction of the photons from the point at which they interacted. All photons not re-emitted are considered to be absorbed. This re-emission probability was adjusted to be 0.9 to obtain agreement with the data. The comparison with the data was made using the number of PMT hits (due to scattered light) outside the Cherenkov cone of cosmic ray muons.

The geometric model of the phototube consists of a 5-inch hemispherical photocathode connected to a cylindrical housing of equal diameter and 18 cm length. Photons impinging upon any tube housing or the wall of the detector are considered to have been absorbed. The number of photons hitting each photocathode, as well as the time of arrival of the earliest photon at each tube is recorded. This represents the signal (unsmereared by PM resolution) expected from each tube since the effects of quantum efficiency, etc., are taken into account in the generation of photons. For some applications the radius of the simulated tubes are scaled up by a factor of three and the number of generated Cherenkov photons reduced by a factor of 9 to increase the speed of the simulation program; the results are insensitive to this procedure.
B.4 Phototube Response Simulation

Smearing of the PMT firing times of the simulated data is performed by repeated sampling of the single PE timing error distributions (Fig. 2.V.Bl). This is measured from laser triggers at low light levels. The distribution is sampled once per photoelectron recorded at the tube. The earliest time of any of the samplings is taken to be the discriminator firing time. This procedure reproduces the statistical walk and the narrowing of the time resolution of the data at high light levels. A correction for the average value of the statistical walk is made which mimics the pulse height/timing correction made to the data. A prepulsing probability of .003 per photoelectron is included, as is PMT dark noise of 2.7 KHertz/tube.

Fig. 3.III.7 displays the point fit timing residuals for the contained events as well as for simulated neutrino events passing the same cuts. The width and tails of the distributions are in excellent agreement, indicating that the timing of the PMT's is being accurately simulated.

Pulse height resolution smearing arises primarily from two sources: (a) Poisson statistics in the number of photoelectrons which arrive at the first dynode of the tube, which has already been taken into account by the random generation and collection of simulated Cherenkov photons,
and (b) statistics, and variations in the surface quality and electron optics of the first few stages of multiplication in the PMT dynode chain. This second effect is simulated by repeated sampling of the single PE pulse height distribution (Fig. B.4). This distribution is measured using laser data at low light levels. The distribution is sampled once per photoelectron recorded at each tube, and the total charge registered is the sum of the samplings.

B.5 Electromagnetic Showers

Electromagnetic shower simulation in the detector makes use of the EGS (Electron Gamma Shower) routines [38] which were developed at SLAC. They incorporate multiple scattering, bremsstrahlung, Moller, Bhabha, and annihilation processes of positrons and electrons, and Compton, pair production, and photoelectric effects for photons. Electrons and gammas are followed, in steps of 1 cm or less, to the Cherenkov threshold kinetic energy of 0.3 MeV and Cherenkov light is generated independently for each segment. Internal energy conservation checks, etc., have been run to ensure the correct operation of our copy of these routines.

Comparison of experiment with EGS simulations, in water, have been performed by Crannell et. al. [49] and have been found to be in excellent agreement as shown in Fig. 2.IV.3.
Fig. B.4  PMT pulse height distribution used in simulation.
B.6 Nuclear Effects in $p + e^+\pi^0$

Nuclear effects in $p + e^+\pi^0$ in oxygen were simulated via an intranuclear cascade model due to T.W. Jones [39]. The underlying assumption of the model is that the nuclear interactions of the $\pi^0$ may be approximated by a series of two-body collisions, with interaction probabilities determined from nuclear density profiles and free particle cross sections. The model is adjusted to obtain agreement with measured $\pi$-nucleus scattering in the energy range of interest, then used to estimate the effects of $\pi^0$ reinteraction following proton decay.

The initial position of the decaying proton is chosen according to nuclear density profiles determined from electron scattering [51]. The results are not sensitive to the details of the distribution within the nucleus; for example, an initial position which is always taken to be at the center of the oxygen nucleus yields essentially the same detection efficiency for $p + e^+\pi^0$. The Fermi motion of the of the decaying nucleon is is chosen randomly from a spherical box distribution (in momentum space) of radius 225 MeV/c. No correlation of the Fermi motion with the decay position in the nucleus is assumed. The effects of binding energy are taken into account only to reduce the energy of the decaying proton ~25 MeV from $\sqrt{m_p^2 + p_{Fermi}^2}$ to $m_p$. The additional 10-20 mev of binding energy (small compared to
the detector energy resolution of 10% at 1 GeV) is neglected. The initial energies of the \( e^+ \) and \( \pi^0 \) are then determined from the two-body decay of a particle with momentum \( p_{\text{Fermi}} \) and total energy 938 MeV.

The \( e^+ \) is presumed to escape the nucleus unscathed. The \( \pi^0 \) is propagated through the nucleus in steps of 0.2\( f \), with energy-dependent cross sections for inelastic and charge exchange scattering, and absorption. The \( \pi^0 \) cross sections are obtained by relation to measured cross sections for charged pions and free protons, except for pion absorption (which cannot occur off of free protons). The absorption is adjusted to conform to the pion-oxygen data interpolated from the pion-nucleus cross sections reported by Ashery [52], as described in Ref. [39].

The cuts in opening angle and energy used in the literature to compare the survival efficiencies for \( p + e^+\pi^0 \) events are somewhat more restrictive than those used in Chapter 4; however, with these cuts applied, the model we use seems to be in good agreement with the central values of other published estimates, as shown in the table below.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Fraction of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \theta &gt; 157^\circ )</td>
</tr>
<tr>
<td></td>
<td>( E_{e^+} + E_{\pi^0} &gt; 0.95 , m_p )</td>
</tr>
<tr>
<td>Longemarre [41]</td>
<td>70%</td>
</tr>
<tr>
<td>Gabriel &amp; Goodman [25]</td>
<td>55%</td>
</tr>
<tr>
<td>T. W. Jones [39]</td>
<td>52%</td>
</tr>
<tr>
<td>Nishimura &amp; Takahashi [53]</td>
<td>( \sim 55% )</td>
</tr>
<tr>
<td>Sparrow [40]</td>
<td>( \sim 30% )</td>
</tr>
</tbody>
</table>
APPENDIX C - EVENT FITTING ROUTINES

C.1 The Point Fit

This Appendix describes the procedure, motivation, and clever tricks behind the 'lattice point fit' which obtains a rejection factor of 30:1 against entering tracks with less than 300 tubes (see Sect. 3.III), at 5 events per second on a Vax 11/780.

Point fits rely on a model of the event as an instantaneous point source of light. The fit has four (3 spatial plus 1 time) degrees of freedom. Conceptually the procedure is as follows:

(1) Pick a point $\bar{x}_o$ inside the detector as a candidate vertex.

(2) Construct [a histogram of] the timing residuals for each tube:

$$\Delta t_i = t_i - (1.33/c)\sqrt{\bar{x}_o - \bar{x}_i}^2 - t_o,$$

which are simply the firing times $t_i$ of each tube after correction for the photon time of flight from the point to the tubes (at positions $\bar{x}_i$), and an arbitrary time offset $t_o$. If you have chosen the correct vertex of an event which was truly an instantaneous point source of light, the histogram should look something like the inherent time
resolution function of the tubes. If the point chosen was far from the vertex, the distribution should be wider.

(3) Find some measure of the 'goodness' of the timing residuals ($\chi^2$, throwing out points followed by $\chi^2$, maximum likelihood, etc.) The details of this procedure are a major source of differences between point fitters.

(4) Choose $t_0$ to maximize the 'goodness' of the residuals. For $\chi^2$ minimization $t_0$ is trivially the mean of the distribution, and for maximum likelihood this involves convoluting the residuals with a weighting function (given by the logarithm of the firing time probability distribution of the PMT), then adjusting $t_0$ to maximize this convolution. This $t_0$ optimization should be repeated each time the fit vertex is moved.

(5) Move the fit vertex $\bar{x}_0$ and loop back to (2). The spatial iteration structure is an important factor in both the speed and reliability of the point fitter. Many schemes are possible here, from 2nd order gradient fitters to the slow but reliable method of laying down a (3-dimensional) search grid.

Problems which arise with $\chi^2$ minimization. The great advantage of $\chi^2$ is that the $t_0$ minimization is quick and trivial ($t_0$ is simply given by the mean of the distribution), but there are a number of drawbacks to this procedure:
(1) Simple $\chi^2$ minimization will be dominated by the non-Gaussian wings (noise, prepulsing, scattered light...) of the timing distribution, and will yield inaccurate results.

(2) Clipping the timing wings (say $\pm 20$ ns. about the mean) before $\chi^2$ minimization improves the situation, except that:

(a) The position of the mean is also sensitive to the (asymmetrical) wings of the timing distribution, and if the cuts are improperly placed they will throw out good data. It is also possible to throw out good data if your first guess is inaccurate.

(b) What do you do about the tubes which are thrown out? Clearly the $\chi^2$ should be punished, somehow, for a fit in which most of the tubes are expelled from the fit - especially if they are a causally connected to the rest of the event.

(c) Do you reinclude tubes which have been previously thrown out, if they fall inside the timing cuts after the fit evolves? This becomes time consuming and can introduce oscillatory behavior into the fits.

The method of maximum likelihood offers a solution to some of these problems in that tubes are gradually ignored by the fit as their timing residuals get large, and smoothly reincluded if the fit evolves so that the residuals become small again. The likelihood is evaluated by convoluting the
timing residual histogram with a weighting function which is essentially the log of the timing distribution of the tube (Fig. C.1). It has a great disadvantage, however, in that the \( t_0 \) optimization is much slower than for \( \chi^2 \): the convolution must be repeated for different values of \( t_0 \) until the likelihood is maximized, which is far slower than simply calculating the mean of the distribution.

**Clever Trick #1** involves approximating the maximum likelihood weighting function (Fig. C.1(d)) by a stepped function (Fig. C.1(e),(f)). In the gross approximation (Fig. C.1(e)) of a square box, the maximum likelihood convolution simply amounts to taking the area of the timing residual histogram which overlaps the box, and then sliding the box around (i.e. varying \( t_0 \)) to maximize the overlap. This already simple procedure can be speeded up still further by observing that, as the box sweeps across the histogram, the number of tubes inside the overlap only changes due to the leading and trailing edges of the box: hence, the maximum of the "convolution" can be obtained by sweeping a pair of pointers (one at each edge of the box) across the timing residual histogram and keeping track of the histogram area between them.

To obtain a better approximation to the nominal maximum likelihood weighting function, one can stack a number of boxes on top of each other (Fig. C.1(f)), with of course one
(a) Tube timing residuals near correct vertex.

(b) Parabolic $\chi^2$ weighting function.

(c) Truncated $\chi^2$ weighting function.

(d) Maximum Likelihood weighting function. 
$log(PMT\ firing\ time\ probability\ distribution)$

(e) Gross Approximation to Maximum Likelihood weighting function.

(f) Stepped Approximation to Maximum Likelihood weighting function for fast $T$-zero optimization.

Fig. C.1. PMT timing distribution, and point fit weighting functions discussed in the text.
additional pair of pointers needed for each box. The point fit 'goodness' $G$ is then defined as the maximum of the convolution of the residual histogram with the weighting function, normalized by dividing by the number of tubes firing in the event. Thus if the event has perfect point-source timing, the residual histogram will be a spike with all the counts in one bin, and the normalized convolution will be 1.0.

In practice I have made no effort to use the exact form of the maximum likelihood weighting function, but have instead chosen a somewhat wider ad hoc approximation (Fig. C.1f) which gives rise to fairly smooth and broad (spatial) maxima in the point fit goodness. Higher spatial resolution can be achieved using a somewhat narrower convolution function, but this seems to complicate the structure of the maxima and to increase the chances that the point-fit will get stuck in a subsidiary (non-global) maximum.

Clever Trick #2 follows from a few observations:

(1) The calculation of the timing residuals requires calculating the distance from the point to each tube which was hit, which requires taking a square root, and the time to take this square root will normally dominate the time taken for the rest of the procedure.

The tube positions are on an approximate rectangular lattice of $91.5 \times 99 \times 105$ cm, and if the fit point is
restricted to lie on the same lattice, then the distance to each tube is only a function of the difference of the "lattice coordinates":

\[ \text{distance} = \text{DIST}(X-X_0, Y-Y_0, Z-Z_0), \]

where the X's, Y's and Z's are integer lattice coordinates (X=0..25, Y=0..17, Z=0..17) and DIST is a three dimensional lookup table. This three dimensional array lookup is much faster than any square root function call. In addition various constants can be absorbed into the lookup table, so that DIST actually contains the time-of-flight (in nsec.) from the point to each tube.

(3) The situation can be improved still further by observing that the 3-dimensional array lookup (timewise, still the dominant part of the calculation) can be made via an equivalent 1-dimensional lookup using the difference of the radix-packed lattice coordinates of the fit point and the firing tubes.

Less clever tricks which are important to speeding up the lattice point fit were as follows:

(a) Assembly language subroutines were written for the 'tight loops' (i.e. the inner DO-LOOPS where the program spends most of its time.) The Vax Fortran compiler is very good, and this alone produced less than a factor of 2 improvement in speed.

(b) The base addresses of several of the arrays could
be absorbed into the data in some of the lookup tables, which resulted in simpler and faster addressing modes on the Vax, and in some cases eliminated whole instructions from the tight loops.

(c) When a sequence of a few instructions has to be repeated many times (as in a tight loop) normally one uses an instruction which loops back to repeat the sequence many times. This loop-back instruction can be a significant part of a loop which is only a couple or few instructions long. This is especially true since processor has to throw away all of the pipelining and instruction-prefetch work that it has done, and start over at the address you jumped back to. The trick to eliminate this is to repeat the basic sequence of instructions several times and then loop back, thereby reducing the loop-back overhead to a small fraction of what it would have been.

The net improvement in speed, from all the tricks described here, was about a factor of 40 in the time required to evaluate the point-fit goodness of a single point inside the detector.

**Spatial Iteration Techniques** are another major source of differences among point fitters. I have had very little success with any method that involved taking a derivative, either analytically or using finite differences. The
problems here are:

(1) For maximum likelihood the analytic derivative is hopelessly complex.

(2) How do you, analytically, take the derivative of the fact that if you move any distance in some direction you are going to throw out some set of tubes? The fit goodness will behave discontinuously and the derivative will be meaningless.

(3) The discontinuities in the fit goodness (which are present in most of the measures of fit goodness described above) play havoc with any attempts to evaluate the derivative thru finite difference methods. Second derivatives are even worse. Finally, it is unclear what step size should be taken to evaluate the derivative.

Following a grid downhill is by far the most successful method I've tried. Conceptually it is simple:

(1) Choose a step size (initially 1-2 meters)

(2) Step ± in X, Y, and Z, and see if the fit improves. It is important to look around in all directions before making a move: for example, if you move to the first point that offers any improvement at all, then the fit may end up charging down a blind alley and getting trapped.

(3) When the fit stops moving, decrease the step size and go back to (2). (This obviously does not apply to the lattice point fitter, where the step size is fixed at 1
lattice spacing).

This is admittedly a brute-force approach to the problem, but a very fast fitter applied in a brute-force manner is apt to be much more reliable than a slow but tricky fitter.

The First Guess is an important element in the speed and reliability of the fitter. If the first guess is inaccurate, the fitter will take a long time to converge to the correct answer and there is a chance that it may get stuck in a subsidiary minimum. Three possibilities for first guesses suggest themselves:

(a) The coordinates of the first 5-10 tubes in an event. At least one of these tubes will be a good approximation for a typical entering track, and the early tube with the best point fit is generally an excellent starting point.

(b) The center of mass of the hit tubes in the event. This will be a reasonable approximation for isotropic contained events. For contained single-track events, this will often be nearly along the track axis, and will again be a reasonable starting point for the fit.

(c) A very coarse (2m-10m) search grid, over the whole volume of the detector, to identify the approximate position of the minimum.
In practice, the best point fit from either (a) or (b) is used as the starting point for the first pass point fit, with method (a) yielding the best result >99% of the time. Method (c) is used with a 2m. search grid as part of the exhaustive single-track fit described in Appendix C.2.

C.2 SINGLE TRACK FIT

This Appendix describes the procedures, motivation, and clever tricks behind the single track fitter which is used to reduce the data from the point fit stage (~3000/day) to the scanning stage (~2.5/day). In the \( p + e^+ \nu^0 \) range from 125-225 lit PMT's, the fitter is 100% effective at rejecting entering background, and no scanning is necessary.

The fitter makes several approximations which are, strictly speaking, valid only for short tracks; however, the performance of the fitter seems to be fairly good for long tracks as well.

This Appendix is divided into several sections:

(i) The general idea behind the fitter.

(ii) Calculation of the penalty weights for including tubes into the fit.
(iii) Projecting the weights back onto the track and evaluating the fit goodness.

(iv) Iteration (adjusting the position and direction of the fit track).

(i) The Basic Idea Behind the Fitter.

The basic idea behind the fitter was derived from maximum likelihood. Tubes falling inside the Cherenkov cone are 'likely' to fire, depending on light attenuation and distance from the track, and those outside the cone are 'unlikely' to fire, depending on light scattering and the distance from the track. The logs of these firing probabilities are used to reward a fit for including a hit tube inside the fit Cherenkov cone, and penalize it for including a missed tube. All of the information, including that from the missed tubes, is considered in the fit.

The procedure for fitting a short track then basically falls into two steps:

(A) Construct a set of penalty weightings for including each tube under the cone. These weightings will be positive for hit tubes and negative for missed tubes, but will depend on tube distance from the (assumed short) track.

(B) Adjust the fit Cherenkov cone so that it picks up as many of the positive weights and as few of the negative weights as it can (see Fig. C.2.1). This is done most efficiently by projecting the weights of the tubes back
Fig. C.2.1. The basic idea behind the single track fitter. Firing PMT's are assigned positive weights and missed tubes negative weights on the basis of a modified maximum likelihood model. The fit track is adjusted to pick up the largest number of positive weights and the smallest number of negative weights inside of the fit cone. Ideal 41° Cherenkov geometry is used to project the penalty weights back onto the fit track. A histogram of these weights is constructed, and this histogram is used to determine the start and stop points of the track.
onto the fit track, and adjusting the track direction and start point so as to produce the largest positive bump in the histogram of these projected-back weights.

The use of a complicated weighting scheme for these penalties is, I believe, necessary for best results. Simpler schemes, such as counting hit and missed tubes inside and outside of the Cherenkov cone, suffer from three sources of noise:

(1) You expect, and you see, a large number of PMT firings from scattered light -- especially near the track. Stripping these away with a timing cut will be only partially successful if the cut is not to reduce the occupancy inside the cone as well. A weighting scheme may be able to de-emphasize the importance of scattered tubes near the track.

(2) There are a large number of missed tubes inside the cone, especially at large distances. Recall that only 50% of the tubes inside the cone will be hit at a distance of 17m from the radiating point on the track. If a tube does not fire when it is close to the track, the goodness of fit should be substantially reduced, but if a tube is missed 20 meters away it doesn't mean much: In short, you need weightings.

(3) The density of tubes per unit solid angle about the vertex is often grossly anisotropic (particularly for
entering and corner clipping tracks) and simple counting schemes will be spuriously influenced. A maximum likelihood approach may do a better job of rejecting this influence, since a nearby wall full of lit tubes will be assigned a lower weight.

(ii) Calculation of the Penalty Weights.

Maximum likelihood provides a general scheme for justifying this procedure as well as calculating the weights. A model of the event is required which predicts the level of illumination at each tube:

\[ \text{NPE}_{\text{direct}} = \exp(-R/25\text{meters}) \times \frac{K_1}{R} \]
\[ = \# \text{ of } \text{PE's} \text{ expected from Cherenkov light, and} \]

\[ \text{NPE}_{\text{scattered}} = \exp(-R/25\text{meters}) \times NPT \times \frac{K_2}{R^2}, \]
\[ = \# \text{ of } \text{PE's} \text{ expected from scattered light,} \]

where:

\[ K_1, K_2 = \text{constants chosen to agree with the data} \]
\[ NPT = \text{number of tubes in the event, and} \]
\[ R = \text{distance from the track to the tube.} \]

The light expected for tubes inside and outside the cone is then

\[ \text{NPE}_{\text{inside}} = \text{NPE}_{\text{direct}} + \text{NPE}_{\text{scattered}} \quad \text{and} \]

\[ \text{NPE}_{\text{outside}} = \text{NPE}_{\text{scattered}}. \]
Which one of these two possibilities is expected will depend on whether the given tube lies under the currently fit Cherenkov cone. Since the probability of a tube being missed is simply $\exp(-NPE)$ and the probability of it being hit is $(1-\exp(-NPE))$, one can calculate the likelihood that a given pattern of tube hits and misses will be produced from a given radiating track. This is just the product of 2048 probabilities, chosen one of 4 ways for each PMT:

<table>
<thead>
<tr>
<th></th>
<th>(A) Tube lies inside fit Cherenkov cone.</th>
<th>(B) Tube lies outside fit Cherenkov cone.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Tube hit in time.</td>
<td>$(1-\exp(\text{NPE}_{\text{inside}}))$</td>
<td>$(1-\exp(\text{NPE}_{\text{outside}}))$</td>
</tr>
<tr>
<td>(2) Tube missed or hit out of time.</td>
<td>$\exp(-\text{NPE}_{\text{inside}})$</td>
<td>$\exp(-\text{NPE}_{\text{outside}})$</td>
</tr>
</tbody>
</table>

In order to maximize the product of these 2048 probabilities, one actually maximizes the sum of the 2048 logs of these probabilities. These logarithms are extremely slow to calculate and, since they must be recalculated for both hit and missed tubes each time the fit track is moved, they would normally make the maximum likelihood approach useless.
Trick #1 is to make a number of 'short track' approximations which are needed in order to bring the calculation of these logs outside of the inner iteration loops of the fit which adjust the track angle. These approximations are:

(1) The timing for all tubes is determined by a point source at the start point of the track. As discussed in Sect. 3.3, this is an excellent approximation for tracks $< 3-5 \text{ m long}$.

(2) The radial distance $R$ in the Cherenkov light intensity, which should properly be the distance from the radiating point on the track (and hence a function of track direction), is approximated by the distance to the (fixed) start point on the track. This is a good approximation for tracks whose lengths are short compared to the distance to the tubes.

(3) The scattered light intensity is proportional to NPT and decreases as $1/R^2$ independent of track angle. This approximation ignores the finite length of the track, the fact that the scattered light travels as directional Cherenkov light before scattering, and the fact that the light scattering is not isotropic.

Under these assumptions, the logarithms in the table above, as well as related factors involving time, are a constant independent of track direction. The only effect of
moving the track direction or adjusting the track length is that individual tubes will jump back and forth between columns (A) and (B) as they are moved into and out of the fit Cherenkov cone. This introduces an effective penalty (= log(column A) - log(column B)) for including a tube inside the fit Cherenkov cone. The penalties will be positive for hit tubes and negative for missed tubes, and are exactly those weightings necessary for the procedure in Fig. C.2.1.

**Inclusion of Timing Information Into the Fit.** We have so far totally ignored timing in considering only hit and missed tubes in an event. Clearly, only those tubes with timing errors of less than ~15 nsec are likely to have been fired from the direct light of the event and all others should be treated basically as missed tubes. Thus instead of treating the T1 information of the tubes as a single bit, I make a bell-shaped interpolation, based on point-source timing errors, between the logarithms of the hit and missed tubes. The bell-shaped interpolation function is of the form \(1/(1+x^2)\), with a width 3 times larger for positive timing errors (late PMT firings). Interpolation values between 0 and 1 are used to select between the weightings for missed and hit tubes, respectively.

This effectively imposes the constraint on the (best) fit that the firing times of the tubes be approximately consistent with a point source at the fit vertex,
since the only way for the fit to collect the positive weights for all of the hit tubes is to locate itself at a vertex which produces small timing errors (and hence a point near the maximum of this bell-shaped interpolation) for each tube. For single tracks, this constrains the best fit to be somewhere along the track axis, with the position along the axis free to be determined by the geometry of the event. For multi-track events this constrains the vertex of the best fit single track in the event to be near the best overall point fit, since all timing errors are calculated from the value of $t_0$ based on the point fit of all tubes in the event. This is important, since it means that a fiducial volume cut made on the vertex of the best single track in an event will be meaningful even for multi-track events. See Fig. 3.IV.3(d).

Note that this interpolation calculation, since it depends only on the point fit timing residuals of the fit vertex, can also be taken out of the fitting loop which adjusts the track angle.

Complications and Simplifications. Adjustments to the maximum likelihood penalty weights above are made to correct for the following effects:

(1) The presence of noise, tube dead time, and dead tubes in the data make it necessary to limit the tube firing probabilities to lie between .005 and .995. This avoids a situation in which the fit likelihood would be dominated by
the data from a few misbehaving tubes.

(2) The timing interpolation affects the penalty weights for hit and missed tubes differently. The positive penalty weights for hit tubes are scaled by a factor of 1.5 as an average correction for this effect.

(3) Events with low numbers of lit PMT's have a tendency to have low fit goodesses (due to the folding over of the Cherenkov cone for stopping tracks). This was corrected for by multiplying the fit goodness Gl by a correction factor which depended on NPT. This correction did not affect the position of the fit single track fit, but only the goodness value Gl returned by the single track fitting routine. This correction was only applied to events with <100 lit PMT's.

The absolute likelihood of the fit hypothesis could in principle be evaluated from a sufficiently detailed model of the event, and could conceivably be used to distinguish between, e.g., one and two track hypotheses. The compromises made in the name of speeding up the fitter, as well as the extreme sensitivity of the absolute likeness to the details of the model, make this essentially hopeless.

(iii) **Projecting the weights back onto the track**

and the evaluation of the fit goodness.

The geometry for projecting the tube weights back onto
the track is illustrated in Fig. C.2.1. The track is parameterized by a starting point \( \overline{X}_0 \) and direction cosine vectors \( \overline{v} \). The vector from the point to each tube is \( u_i \). The desired quantity is:

\[
S_i = \frac{\mathbf{u}_i \cdot \overline{v}}{\cot(\theta_c)} \sqrt{u_i^2 - \left(u_i \cdot \overline{v}\right)^2}
\]

which is simply the position along the track for illumination of the tube by a photon emitted at the Cherenkov angle of 42°, as measured from the point \( \overline{X}_0 \).

A histogram of the penalty weights is used to determine the starting and stopping point of the track as shown in Fig. C.2.1. The construction of this histogram involves the calculation of \( S_i \) for every tube (including missed tubes) and is very time consuming.

**Clever Trick #2** is to observe that since we are fitting short tracks with a vertex in the vicinity of \( \overline{X}_0 \), we do not need to construct the weighting histogram for tubes which project back to the track a long distance from \( \overline{X}_0 \). The PM tubes are divided up into 8 x 8 'patches' as shown in Fig. 2.III.B2.2. A quick test (based on the projection of a tube near the center of each patch) allows one to skip over entire patches which will not contribute to the relevant part of the weighting histogram. This avoids doing any calculation for more than 1/2 of the patches in a typical event.
Clever Trick #3 is to observe that the distance $S$ is a smoothly varying function of the tube position for patches which are far from the fit track, and hence can be Taylor expanded as a polynomial

$$S(m,n) = A m^2 + B n^2 + C m n + D m + E n + F$$

where $m=1...8$, $n=1...8$ are the integer coordinates of the tubes within each patch. The expansion is performed about the 37th tube (near the center) in each patch. RMS error of the expansion is $<10$ cm for patches which are more than a few meters from the track. The calculation is performed directly for the few tubes in an event which lie closer than this.

Clever Trick #4. This quadratic polynomial for $S$ does not have to be completely recalculated for each tube, but can instead be generated by stepping a difference equation (i.e. the relationship between $S(m,n)$ and $S(m,n+1)$, etc.) This reduces the calculation required at each tube to two register-to-register integer add instructions (calculation of $S$), and one memory-to-memory add instruction (filling the weighting histogram). Evaluating the goodness for a single adjustment of the track direction takes less than 10 msec, compared with $\sim 300$ msec for a direct calculation involving floating point dot products and square roots, etc.

Evaluation of the fit goodness from the projected-back
penalty weighting histogram proceeds as follows:

(1) An initial track length is assumed, using the approximation:

\[ \text{TRACK LENGTH (cm)} = 3.1 \times \text{(# of Lit PMT's in event, NPT)} \]

(2) A "coincidence window" of length equal to this track length slides across the the weighting histogram in order to find the biggest bump of about the right size. The start point of the box is constrained to be \( \pm 1 \text{m} \) from the nominal start point \( S=0 \) (this corresponds to vertex at the point \( X_0 \)).

(3) Small \( (\pm 1 \text{m}) \) adjustments are made to the length of the fit track by independently adjusting the start and stop points to maximize the fit goodness. This adjustment is important, since top-entering muon events which are point fit near the top of the fiducial volume will often have the best-fit vertex moved out of the fiducial volume by this adjustment (see Fig. 3.IV.1).

(4) The overall single track fit goodness \( G_1 \) is defined as the sum of the weights collected under the best-fit track, divided by NPT. The scaling of \( G_1 \) is arbitrary, but is chosen such that values of 3.5-5 are typical for clean single muon events in the detector.

(iv) Iteration Structure of the Single Track Fitter.

Adjusting the track angle at each point is done by
scanning across a 3 x 3 angular grid in the vicinity of the current fit track. If the scan results in a better fit being obtained away from the center point of the grid, it is repeated about the new best fit direction; if not, the grid size is reduced by a factor of 2 and the scan repeated. The grid size is initially 0.1 radians and the fit is terminated at a grid size of 0.1/16 radian ~ 0.36 degrees. Typically ~50 different directions are examined at each point in ~0.5 seconds. The initial guess for the fit direction is the average direction cosine from the vertex to the hit tubes.

The quick single track fit uses as candidate vertex points: (a) The best lattice point fit position found by the point fit. (b) The positions of the first 8 tubes to fire in an event. Since the first 1-2 PMT's which fire in an event are likely to have fired from PMT dark noise, the first 8 tubes are searched in the sequence (3,4,2,5,1,6,7,8). (c) The first position tried is the position of the tube(out of the first 5 tubes to fire) which provided the best point fit. This increases the speed at which tracks can be quickly rejected. Specifically, rejecting an event requires a value of fit goodness Gl>2.5 for a vertex position <75cm in from the planes of the PMT's. If a good fit to an entering track is not yet found, the quick single track fitting program then tries displacing the track laterally by ±150 cm in each coordinate normal to the fit track direction. If this fails to identify a good fit
to an entering track, the quick single track fit has failed and the event is saved.

The exhaustive single track fit proceeds in 2 steps. The first step is the identification of the 50 lattice sites (on a 3-dimension 1m grid inside the detector) which provide the best point fit timing. This search uses the lattice point fitter described in Appendix C.1. These 50 lattice sites are then used as vertex hypotheses for the single track fitter. If at any point in the search a good fit is found to an entering track, the event is rejected. In addition, events are rejected if the best-fit vertex fails fiducial volume cuts which become increasingly restrictive as the fit proceeds: The best fit vertex after 5, 10, and 25 vertex hypotheses is required to lie inside cuts 50cm, 75 cm, and 100cm in from the planes of the PMT's. At the end of the exhaustive search on a 1m lattice, small transverse displacements (± 50 cm in each coordinate) are applied to the fit track in an attempt to improve the fit goodness. At the end of the exhaustive single track fit, the fiducial volume cuts described in Sect. 3.IV are imposed on the best fit vertex.

If at any point it is decided to reject an event, the z-direction cosine of the best-fit track is examined. If the fit track direction is found to be upward-going, the event is flagged ("upward-going over-ride") and the event is
unconditionally saved as described in Sect. 3.VI.

Evaluating each vertex hypothesis takes ~1 sec VAX 11/780 CPU time, so that the exhaustive search takes about 1 minute for an event which is ultimately saved.

C.3 TRACK GONIOMETER

This Appendix describes the details of the fitting program used to determine the opening angles between two-track events. The fitter performance is best for events with tracks which illuminate more than 40 PMT's apiece and which have opening angles greater than 90° so that the Cherenkov cones do not overlap. These requirements are easily met for \( p + e^+\pi^0 \) decays in the fiducial volume and for which the \( \pi^0 \) has not been absorbed or very badly scattered. The fitter makes no effort to resolve the separate showers from the \( \pi^0 \) in \( p + e^+\pi^0 \) decays.

The vertex position in the fit is taken to be the point fit vertex described in Sect. 3.III and Appendix C.1 and is not adjusted during the fit. The average 3-dimensional error of the point fit is 60 cm (Fig. 3.IV.5) for simulated \( p + e^+\pi^0 \) events.

The strongest single track in the event is identified by searching through the 4π solid angle surrounding the event vertex as described below. All phototubes lying within 50° of the event vertex are then deleted from the event, and the
procedure is then repeated to find the second strongest track. For single track events, essentially all tubes will be deleted after the first track is identified, and the number of PMT's associated with the second track will be small enough to reject the event as a 2-track candidate.

The $4\pi$ search through solid angle to identify a track proceeds in two steps. Firstly, 250 points evenly distributed in solid angle are checked to see which provides the best-fit track direction. This direction is then used as a starting point for a search on a $3 \times 3$ angular grid about the current best-fit track position. The grid search is iterated 4 times with successive mesh sizes of 0.1, 0.05, 0.025, and 0.0125 radians. The final mesh size of 0.7° is significantly smaller than the intrinsic resolution of the fit.

The goodness of fit of a track in a given direction from the vertex is evaluated as

$$\text{goodness} = \frac{1}{\text{NPT}} \sum_{i=1}^{\text{NPT}} f_1(\Delta G_i) f_2(\Delta t_i)$$

where

- $\text{NPT} =$ number of lit tubes in an event,
- $\Delta G_i =$ geometrical error of lit tube $i$
  $$= \cos(\text{angle between tube and track}) - \cos(\theta_c=41^\circ)$$
\[ \Delta t_i = \text{point fit timing error of tube } i \]
\[ = \text{tube firing time corrected} \]
\[ \text{for photon time of flight} \]
\[ \text{and fit event starting time.} \]

\( f_1 \) and \( f_2 \) are approximately bell-shaped weighting functions for the geometrical and timing errors of each tube (shown in Fig. C.3). The contribution of the fit goodness of a tube is given by \( f_1 \cdot f_2 \) and hence will be large only if the geometrical error is small (the tube lies near the Cherenkov angle from the fit track) and the PMT timing error is small (the tube is unlikely to be fired from noise or scattered light).
Fig. C.3. Geometrical weighting function $f_1$ and time weighting function $f_2$ used in the two-track fitter.
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