

### Balancing a pencil

Consider a pencil that stands upright on its tip and then falls over. Let's idealize the pencil as a mass  $m$  sitting at the end of a massless rod of length  $\ell$ .<sup>1</sup>

- (a) Assume that the pencil makes an initial (small) angle  $\theta_0$  with the vertical, and that its initial angular speed is  $\omega_0$ . The angle will eventually become large, but while it is small (so that  $\sin \theta \approx \theta$ ), what is  $\theta$  as a function of time?
- (b) You might think that it would be possible (theoretically, at least) to make the pencil balance for an arbitrarily long time, by making the initial  $\theta_0$  and  $\omega_0$  sufficiently small.

It turns out that due to Heisenberg's uncertainty principle (which puts a constraint on how well we can know the position and momentum of a particle), it is impossible to balance the pencil for more than a certain amount of time. The point is that you can't be sure that the pencil is initially both at the top *and* at rest. The goal of this problem is to be quantitative about this. The time limit is sure to surprise you.

Without getting into quantum mechanics, let's just say that the uncertainty principle says (up to factors of order 1) that  $\Delta x \Delta p \geq \hbar$  (where  $\hbar = 1.06 \cdot 10^{-34}$  Js is Planck's constant). The precise implications of this are somewhat vague, but we'll just take it to mean that the initial conditions satisfy  $(\ell\theta_0)(m\ell\omega_0) \geq \hbar$ .

With this condition, find the maximum time it can take your solution in part (a) to become of order 1. In other words, determine (roughly) the maximum time the pencil can balance. (Assume  $m = 0.01$  kg, and  $\ell = 0.1$  m.)

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<sup>1</sup>It actually involves only a trivial modification to do the problem correctly using the moment of inertia and the torque. But the point-mass version will be quite sufficient for the present purposes.