

Week 33 (4/28/03)

**Ball rolling in a cone**

A ball (with moment of inertia  $I = (2/5)MR^2$ ) rolls without slipping on the inside surface of a fixed cone, whose tip points downward. The half-angle at the vertex of the cone is  $\theta$ . Initial conditions have been set up so that the ball travels around the cone in a horizontal circle of radius  $\ell$ , with the contact points (the points on the ball that touch the cone) tracing out a circle (not necessarily a great circle) on the ball.

What should the radius of the circle of these contact points be, if you want the sphere to travel around the cone as fast as possible? (You may work in the approximation where  $R$  is much less than  $\ell$ . Also, assume that the coefficient of friction between the ball and the cone is arbitrarily large.)