Week 59 (10/27/03)

Getting way ahead

A rocket with proper length L accelerates from rest, with proper acceleration g (where $gL \ll c^2$). Clocks are located at the front and back of the rocket. If we look at this setup in the frame of the rocket, then the general-relativistic time-dilation effect tells us that the times on the two clocks are related by $t_f = (1 + gL/c^2)t_b$. Therefore, if we look at things in the ground frame, then the times on the two clocks are related by

$$t_f = t_b \left(1 + \frac{gL}{c^2}\right) - \frac{Lv}{c^2} \, ,$$

where the last term is the standard special-relativistic lack-of-simultaneity result. Derive the above relation by working entirely in the ground frame.

Note: You may find this relation surprising, because it implies that the front clock will eventually be an arbitrarily large time ahead of the back clock, in the ground frame. (The subtractive Lv/c^2 term is bounded by L/c and will therefore eventually become negligible compared to the additive, and unbounded, $(gL/c^2)t_b$ term.) But both clocks are doing basically the same thing relative to the ground frame, so how can they eventually differ by so much? Your job is to find out.