Comparing the numbers

The numbers 1 through $N$ are put in a hat. You and $N - 1$ other people each pick a number. You then compare your number with the others, one at a time, until you find one that is smaller than yours. This procedure is repeated many times. How many numbers, on average, will you have to check in order to find one that is smaller than yours? (Ignore the situations where you have the number “1”.) Consider two cases:

(a) You ask the other people randomly. That is, at all times you have equal probabilities of asking each person. This could be arranged, for example, by demanding that you have a very bad memory, so that you may ask a given person more than once.

(b) You have a good memory. In other words, you don’t ask a given person more than once.