Solution

Week 13 (12/9/02)

Unchanged velocity

Our strategy will be to produce (and equate) two different expressions for the total change in the angular momentum of the ball (relative to its center). The first comes from the effects of the friction force on the ball. The second comes from looking at the initial and final motion.

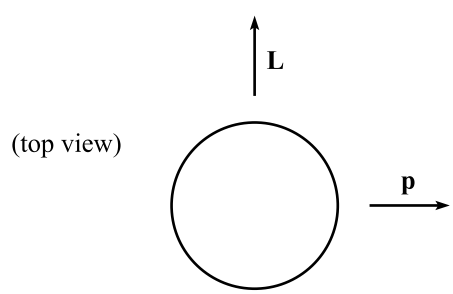
To produce our first expression for $\Delta\mathbf{L}$, note that the normal force provides no torque, so we may ignore it. The friction force, $\mathbf{F}$, from the paper changes both $\mathbf{p}$ and $\mathbf{L}$, according to,

$$\begin{aligned}\Delta\mathbf{p} &= \int \mathbf{F}\,dt,\\ \Delta\mathbf{L} &= \int \boldsymbol{\tau}\,dt = \int (-R\hat{\mathbf{z}})\times\mathbf{F}\,dt = (-R\hat{\mathbf{z}})\times\int\mathbf{F}\,dt. \end{aligned}\tag{1}$$

Both of these integrals run over the entire slipping time, which may include time on the table after the ball leaves the paper. In the second line above, we have used the fact that the friction force always acts at the same location, namely $(-R\hat{\mathbf{z}})$, relative to the center of the ball. The two above equations yield

$$\Delta\mathbf{L} = (-R\hat{\mathbf{z}})\times\Delta\mathbf{p}. \tag{2}$$

To produce our second equation for $\Delta\mathbf{L}$, let's examine how $\mathbf{L}$ is related to $\mathbf{p}$ when the ball is rolling without slipping, which is the case at both the start and the finish. When the ball is not slipping, we have the following situation (assume the ball is rolling to the right):



The magnitudes of p and L are given by

$$\begin{aligned} p &= mv,\\ L &= I\omega = \frac{2}{5}mR^2\omega = \frac{2}{5}Rm(R\omega) = \frac{2}{5}Rmv = \frac{2}{5}Rp, \end{aligned}\tag{3}$$

where we have used the non-slipping condition, $v = R\omega$. (The actual $I = (2/5)mR^2$ value for a solid sphere will not be important for the final result.) It is easy to see that the directions of $\mathbf{L}$ and $\mathbf{p}$ can be combined with the above $L = 2Rp/5$ scalar relation to give

$$\mathbf{L} = \frac{2}{5}R\hat{\mathbf{z}}\times\mathbf{p}, \tag{4}$$

where $\hat{\mathbf{z}}$ points out of the page. Since this relation is true at both the start and the finish, it must also be true for the differences in $\mathbf{L}$ and $\mathbf{p}$. That is,

$$\Delta\mathbf{L} = \frac{2}{5}R\hat{\mathbf{z}} \times \Delta\mathbf{p}. \tag{5}$$

Eqs. (2) and (5) give

$$\begin{aligned} (-R\hat{\mathbf{z}}) \times \Delta\mathbf{p} &= \frac{2}{5}R\hat{\mathbf{z}} \times \Delta\mathbf{p} \\ \Longrightarrow \qquad 0 &= \hat{\mathbf{z}} \times \Delta\mathbf{p}. \end{aligned} \tag{6}$$

There are three ways this cross product can be zero:

- $\Delta\mathbf{p}$ is parallel to $\hat{\mathbf{z}}$. But it isn't, since $\Delta\mathbf{p}$ lies in the horizontal plane.
- $\hat{\mathbf{z}} = 0$. Not true.
- $\Delta\mathbf{p} = 0$. So this must be true. Therefore, $\Delta\mathbf{v} = 0$, as we wanted to show.