Solution

Week 30 (4/7/03)

Difference of Powers

A value of 26 is obtainable with $m = n = 1$. By considering the remainder when $33^m - 7^n$ is divided by certain numbers, we will show that no value smaller than 26 is possible. We will use the "mod" notation for convenience, where $a \equiv b \pmod{c}$ means that a leaves a remainder of b when divided by c.

- Consider divisions by 16. We have $33 \equiv 1 \pmod{16}$, and $7^n \equiv 7$ or $1 \pmod{16}$ because $7^2 \equiv 1 \pmod{16}$. Therefore, $33^m - 7^n \equiv 0$ or $10 \pmod{16}$. So the only possible answers to the problem are 0, 10, 16, and 26.
- Now consider divisions by 3. We have $33 \equiv 0 \pmod{3}$, and $7 \equiv 1 \pmod{3}$. Therefore, $33^m - 7^n \equiv 2 \pmod{3}$. This leaves 26 as the only possibility.