Solution

Week 39 (6/9/03)

Viewing the spokes

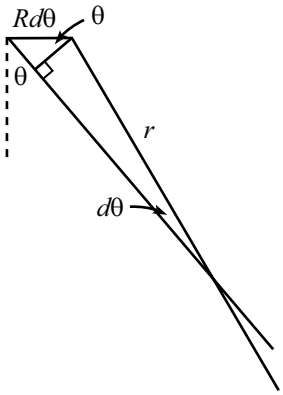
The contact point on the ground does not look blurred, because it is instantaneously at rest. But although this is the only point on the wheel that is at rest, there will be other locations in the picture where the spokes do not appear blurred.

The relevant property of a point in the picture where a spoke does not appear blurred is that the point lies on the spoke throughout the duration of the camera's exposure. (The point in the picture need not, however, actually correspond to the same atom on the spoke.)

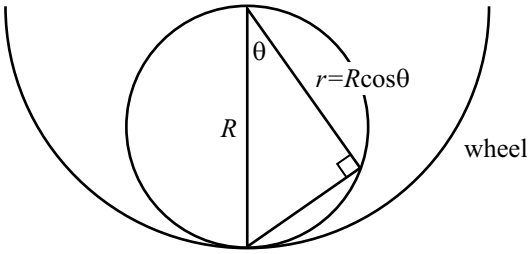
At a given time, consider a spoke in the lower half of the wheel. A short time later, the spoke will have moved (via both translation and rotation), but it will intersect its original position. The spoke will not appear blurred at this intersection point. We must therefore find the locus of these intersections. We can do this in two ways.

First method: Let R be the radius of the wheel. Consider a spoke that makes an angle of θ with the vertical at a given time. If the wheel then rolls through a small angle $d\theta$, the center moves a distance $R\,d\theta$. The spoke's motion is a combination of a translation through this distance $R\,d\theta$, plus a rotation through the angle $d\theta$ (around the top end).

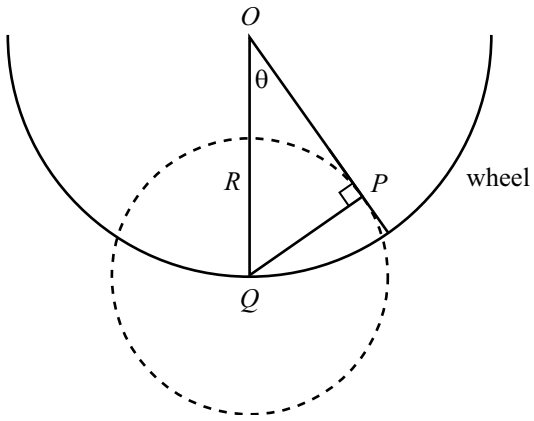
Let r be the radial position of the intersection of the initial and final positions of the spoke. In the following figure, we can write down two expressions for the short segment drawn perpendicular to the initial position of the spoke.



The two expression are $(R\,d\theta)\cos\theta$, and $r\,d\theta$ (to first order in $d\theta$). Equating these gives $r = R\cos\theta$. This describes a circle whose diameter is the lower vertical radius of the wheel, as shown below.



Second method: Since the wheel's contact point with the ground is instantaneously at rest, the wheel may be considered to be instantaneously rotating around this point. This means that every atom in the wheel (both in the spokes and the rim) instantaneously traces out the arc of a circle centered at the contact point. For the point P in the figure below, this circle is the dotted one drawn (with Q being the contact point). A spoke will not appear blurred at the point where this circular motion is along the direction of the spoke. That is, a spoke will not appear blurred at the point where the dotted circle is tangent to the spoke, as shown.



We are therefore concerned with the locus of all points P such that the segments PQ and PO are perpendicular. As seen in the previous diagram above, this locus is the circle whose diameter is the lower vertical radius of the wheel.