Solution

Week 45 (7/21/03)

Sliding along a plane

The normal force from the plane is $N = mg\cos\theta$. Therefore, the friction force on the block is $\mu N = (\tan\theta)N = mg\sin\theta$. This force acts in the direction opposite to the motion. The block also feels the gravitational force of $mg\sin\theta$ pointing down the plane.

Because the magnitudes of the friction force and the gravitational force along the plane are equal, the acceleration along the direction of motion equals the negative of the acceleration in the direction down the plane. Therefore, in a small increment of time, the speed that the block loses along its direction of motion exactly equals the speed that it gains in the direction down the plane. Letting v be the speed of the block, and letting v_y be the component of the velocity in the direction down the plane, we therefore have

$$v + v_y = C, \tag{1}$$

where C is a constant. C is given by its initial value, which is $V + 0 = V$. The final value of C is $V_f + V_f = 2V_f$ (where V_f is the final speed of the block), because the block is essentially moving straight down the plane after a very long time. Therefore,

$$2V_f = V \qquad \Longrightarrow \qquad V_f = \frac{V}{2}\,. \tag{2}$$