The strategy here will be to compute the moment of inertia by using scaling arguments, along with the parallel-axis theorem. To do this, we will need to compare the $I$ for our triangle of side $\ell$ to that of a triangle of side $2\ell$. So let us scale up our triangle by a factor of 2 and examine what happens to the integral $I = \int r^2 \, dm$. We get a simple factor of $2^2$ from the $r^2$, but happens to the $dm$?

A solid triangle would yield a factor of $2^2 = 4$ in the $dm$ (since area is proportional to length squared), but our fractal object is a bit different. The mass scales in a strange way. Doubling the size of our triangle increases its mass by a factor of only 3. This is true because the doubled triangle is simply made up of three of the smaller ones, plus an empty triangle in the middle. Thus, the $dm$ picks up a factor of 3, and so the $I$ for a fractal triangle of side $2\ell$ is $4 \cdot 3 = 12$ times that of a fractal triangle of side $\ell$ (where the axes pass through any two corresponding points).

In what follows, we’ll use pictures to denote the $I$’s of the fractal objects around the dots shown. In terms of these pictures, we have,

\[
\begin{align*}
2\ell & = 12 \triangle

\triangle & = 3(\blacklozenge)

\blacklozenge & = \triangle + m\left(\frac{\ell}{\sqrt{3}}\right)^2
\end{align*}
\]

The first line comes from the scaling argument, the second is obvious (moments of inertia simply add), and the third comes from the parallel-axis theorem (you can show that the distance between the dots is $\ell/\sqrt{3}$). Equating the right-hand sides of the first two equations, and then using the third to eliminate $\blacklozenge$, gives

\[
\triangle = \frac{1}{9} ml^2
\]

**Remarks:** This result is larger than the $I$ for a uniform triangle (which happens to be $ml^2/12$), because the mass is generally further away from the center in the fractal case.

When we increase the side length of our fractal triangle by a factor of 2, the factor of 3 in the $dm$ is between the factor of $2^1 = 2$ relevant to a one-dimensional object, and the factor of $2^2 = 4$ relevant to a two-dimensional object. So in some sense our object has a dimension between 1 and 2. It is reasonable to define the dimension, $d$, of an object as the number for which $f^d$ is the increase in “volume” when the dimensions are increased by a factor $f$. For our fractal triangle, we have $2^d = 3$, and so $d = \log_2 3 \approx 1.58$. 